Guidelines for building a realistic algorithmic trading market simulator for backtesting while incorporating market impact

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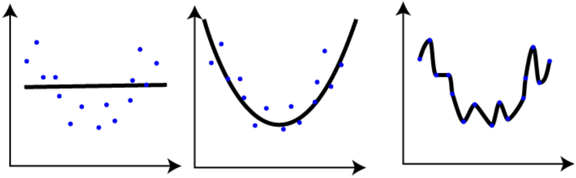
Are we really that different from robots?



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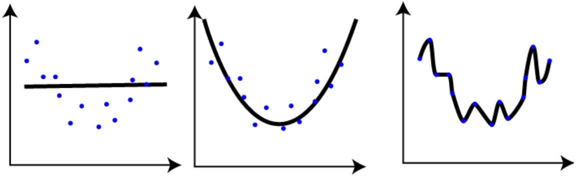

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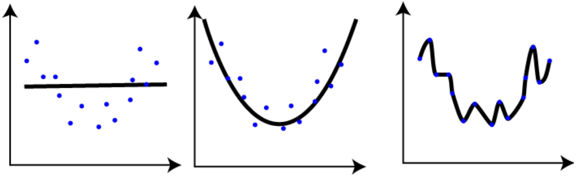

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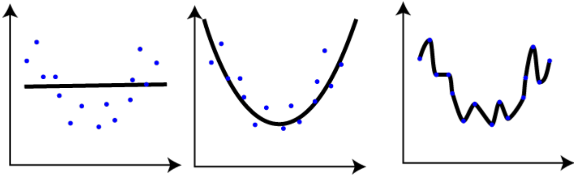

Examples in Finance: backtesting, statistics of order, traders compensation methodology, sales and structured products, technical analysis etc ...

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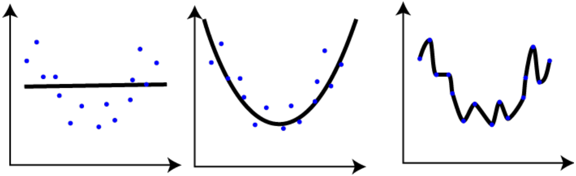

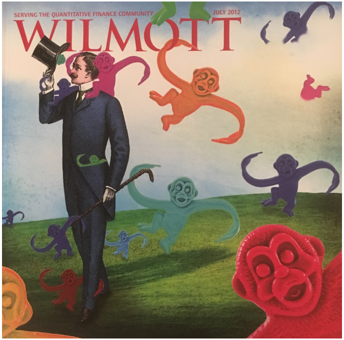

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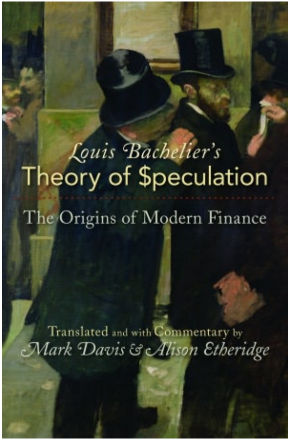
Two simple questions for two simple definitions

Definition (Top-Down Vs Bottom-Up)

Top-Down (TD): Any **stochastic** quantitative approach which assumes that the **market is random** (or close to random) but for which we can create **interesting dynamic strategies**.



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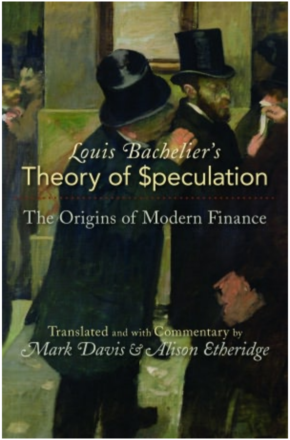

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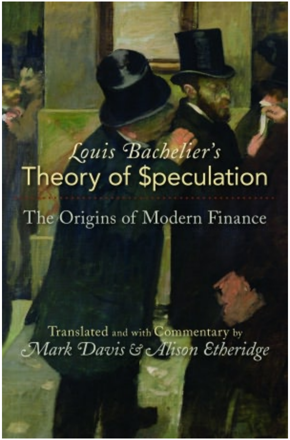
We have tried to understand the financial markets using the top down approach for **100 years with (very) partial success**. Can we look at it in a **different angle**?



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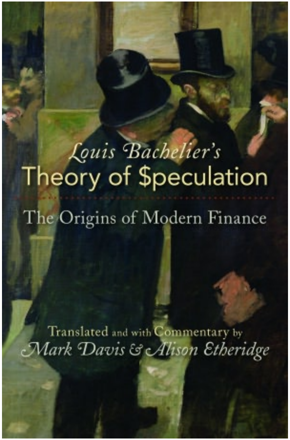
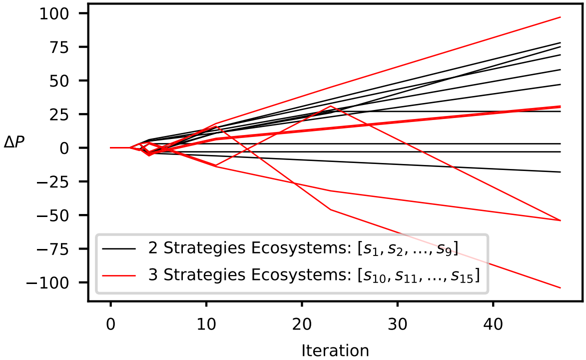


Can we use this new angle to **"solve"** the market?

The Scientific Method for “solving” the market

Question: What do we mean by “solve”?

The Scientific Method for “solving” the market



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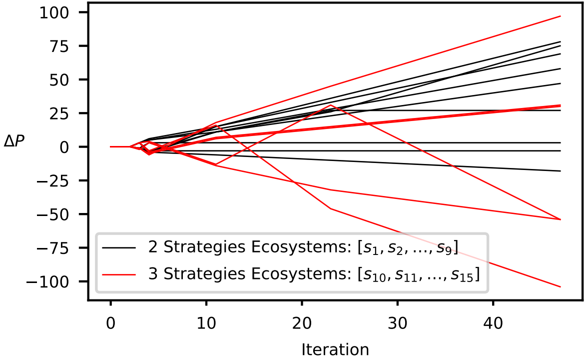

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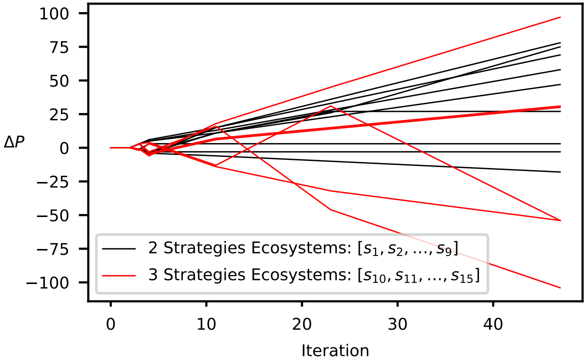
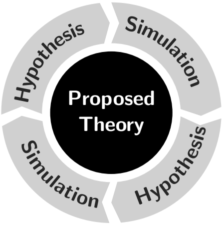


Scientific Method: A good theory can be simulated but simulations can also help bring intuition on what the theory might be [27].



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- **Quantitative Finance was instructed to change** [15]: From **models assuming data to data reassuming the models**,
- The rise of Machine Learning at the expense of traditional probabilistic models [4] though traditional Quant Finance was always enriched by STEM fields (Physics, Mechanics etc ... **what about Biology?**).

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The Rise of Big Data

- **What is Big Data**: lots of anecdotal claims about how big is Big Data [8, 1, 14] but the term refers more to the concept of "**datafication**" (increase in size ≠ better confidence interval but rather change in perspective).

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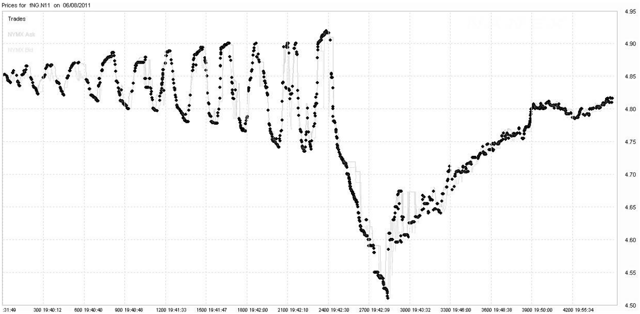

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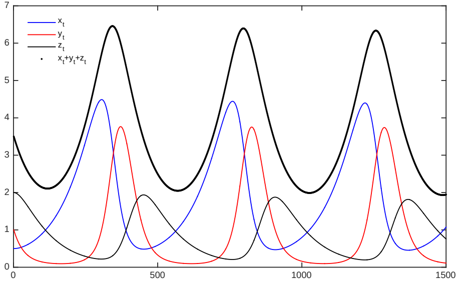
- The **Flash Crashes** (eg: [23]) calls for a **modelling revolution** [5, 11, 6] (**BU vs. TD**): the Brownian motion assumption to model markets is increasingly difficult to defend

Predator/Prey models

biological ecosystems **predator/prey** (PP) models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between $x(t)$, $y(t)$ and $z(t)$ is **deterministic**:

$$\begin{cases} \frac{dx(t)}{dt} & = ax(t) - bx(t)y(t) \\ \frac{dy(t)}{dt} & = -cy(t) + dx(t)y(t) - ey(t)z(t) \\ \frac{dz(t)}{dt} & = -fz(t) + gy(t)z(t) \end{cases}$$



Predator/Prey models

We can make the **hypotheses that the economical cycles** or oscillation in prices are due to the same type of disruptions that can occur in biological ecosystems **predator/prey** (PP) models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between $x(t)$, $y(t)$ and $z(t)$ is **deterministic**:

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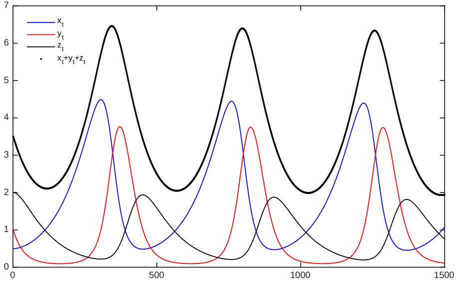

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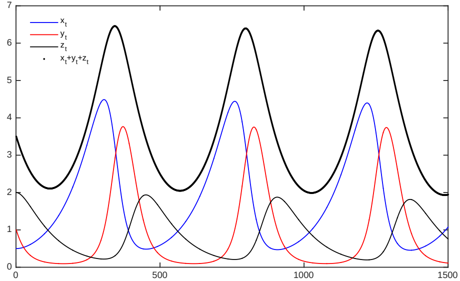
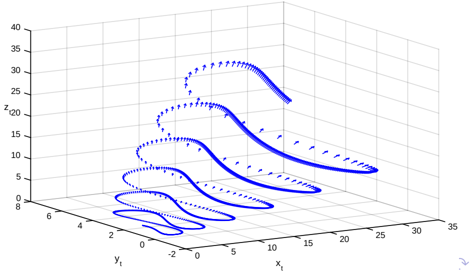


Answering if an **ecosystem** (or by extension financial market) composed of 3 strategies is **stable** would come to studying the Jacobian matrix J. If if all **eigenvalues** of $J(x, y, z)$ have negative real parts then our system is asymptotically **stable**. Though simplistic, the model can easily be expanded to more **complex ecological niches**.

$$J(x, y, z) = \begin{bmatrix} a - by & -xb & 0 \\ yd & -c + dx - ez & -ye \\ 0 & -zg & -f + gy \end{bmatrix}$$



Evolutionary Dynamics

Axelrod's [2, 3] **computer tournament** and **Nowak**'s work on Evolutionary Dynamics (ED) [24] on **invasion** need to influence 21st century Quantitative Finance especially the BU approach.

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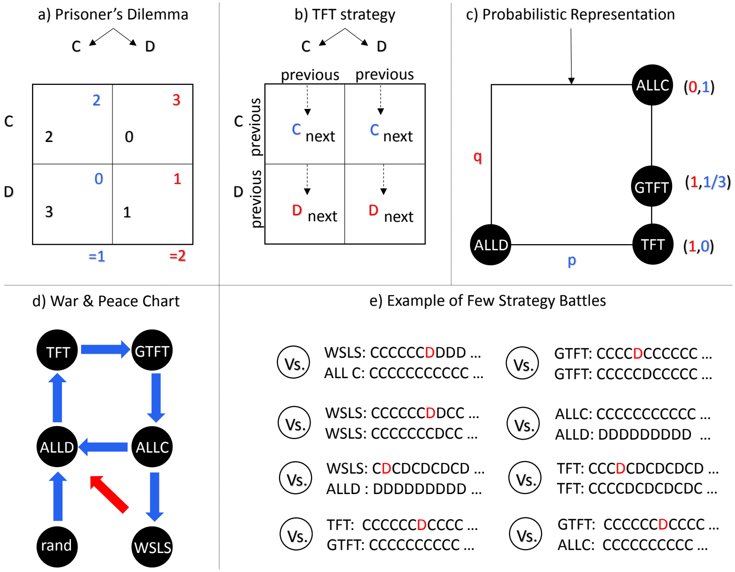

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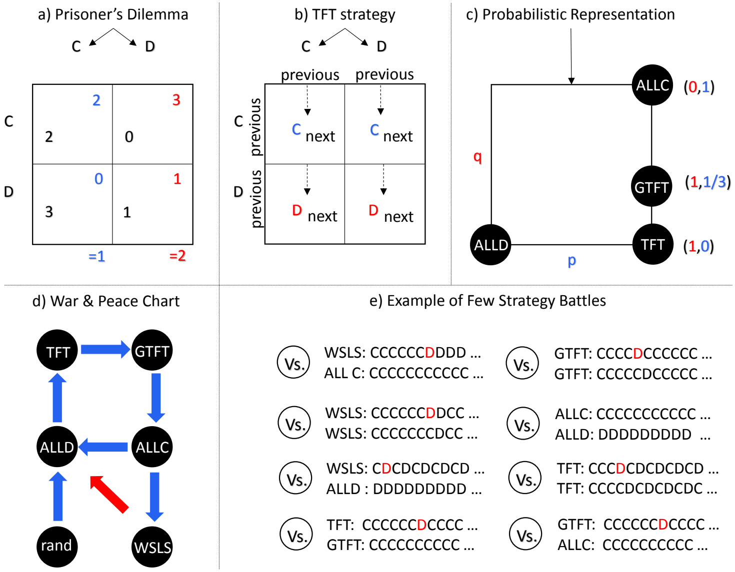

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The methodology in ED is interesting because the **strategies** are both **systematic & interacting** with each other (like it is the case in algo trading).

Generative Adversarial Networks for Electronic Trading

- As ML "translation" of ED and PP models, we have **Generative Adversarial Networks** (GANs) [12], introduced in 2014, usually **involve** a system of **two neural networks** competing in a zero-sum game settings. This process continues as long as needed since the lack of **data is no longer a problem**.

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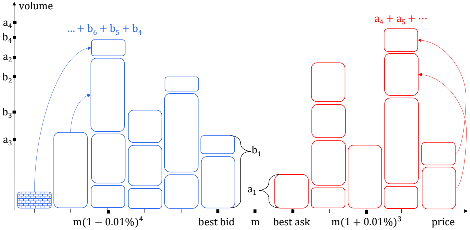

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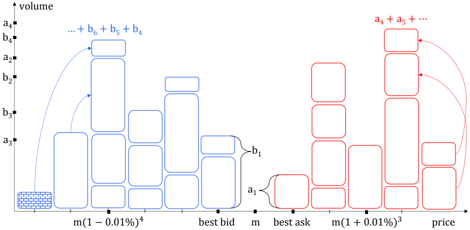
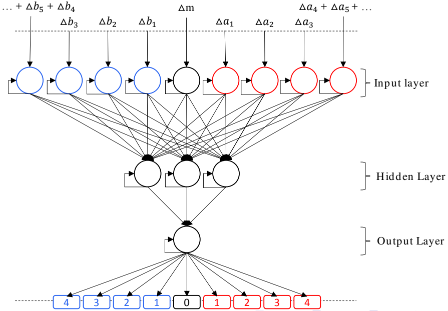
- The **communication tool** is the **order book** and the game is **not zero-sum game** in our research. The strategies are in **High Frequency Financial Funnel** (HFFF) format [20].





High Frequency Financial Funnel & Classic Strategies

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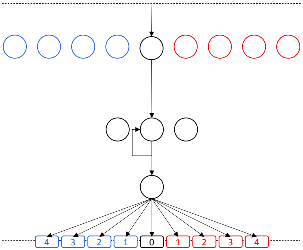

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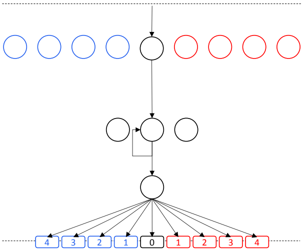

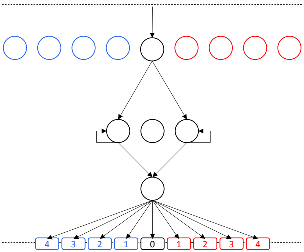

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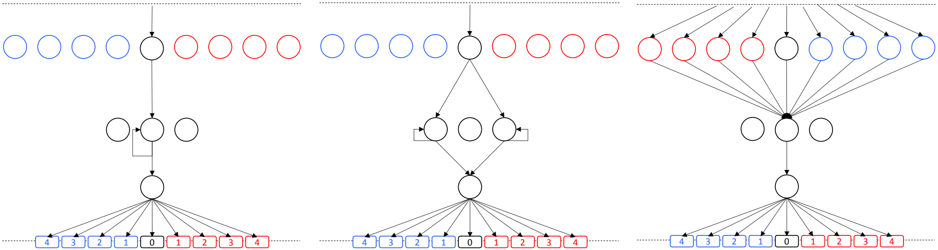

HFFF can model financial strategies:

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3. MLR

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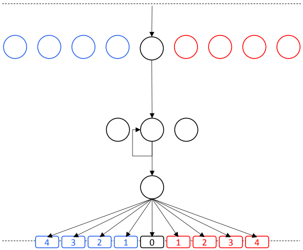

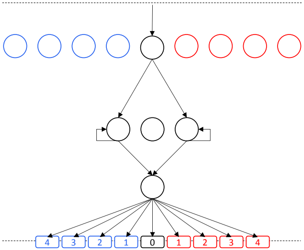

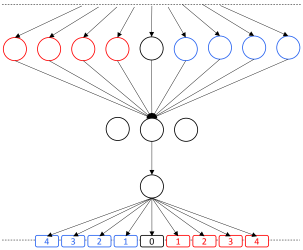

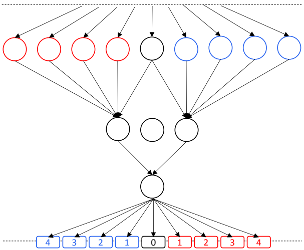

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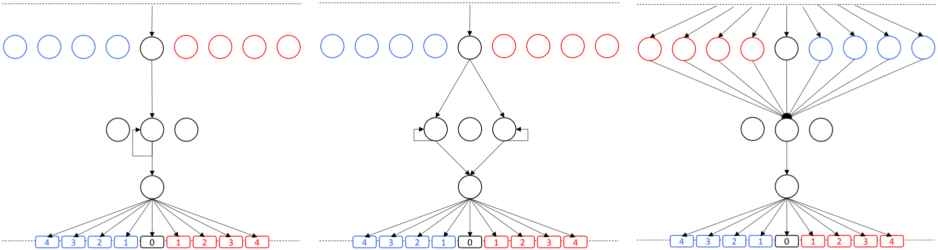

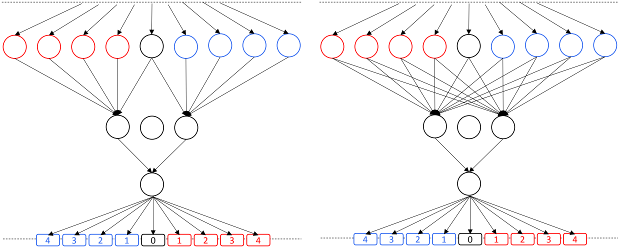

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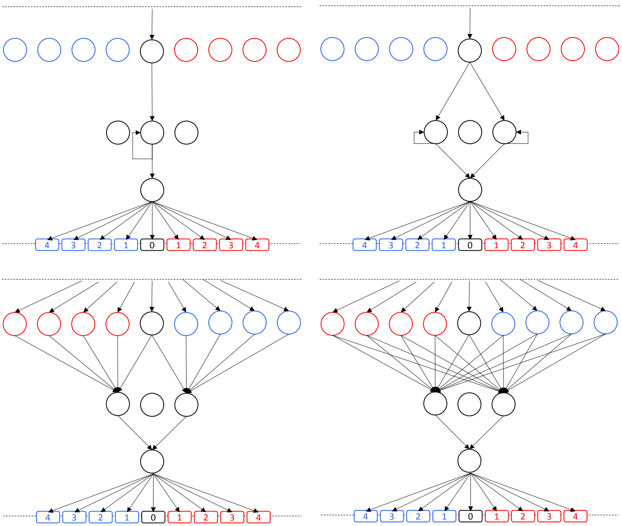

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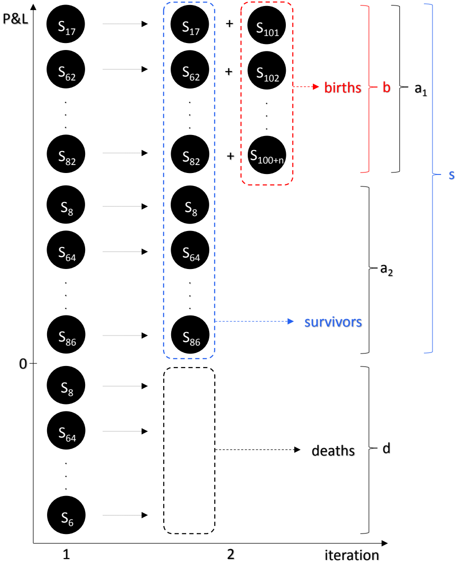
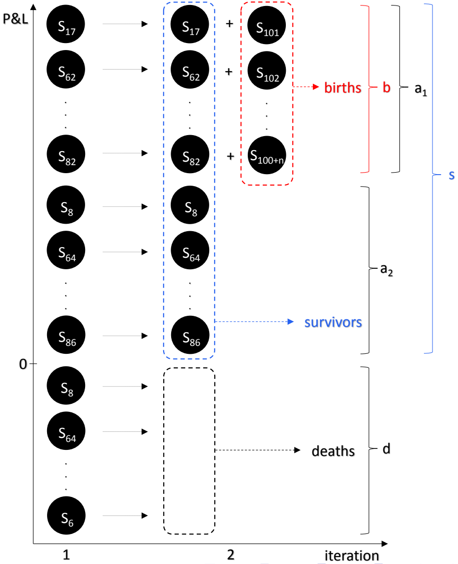
Architecture Complexity and **strategy sophistication** explains the **incentive for Deep Learning** (DL).
Paradoxically we witness **potential for regularization** as the network becomes more complex.

Genetic Algorithm & NN Complexity



Genetic Algorithm & NN Complexity

NN **depth & breadth** should contribute in **increasing the learning potential**.



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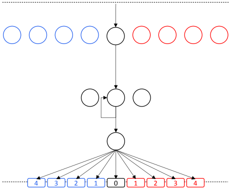
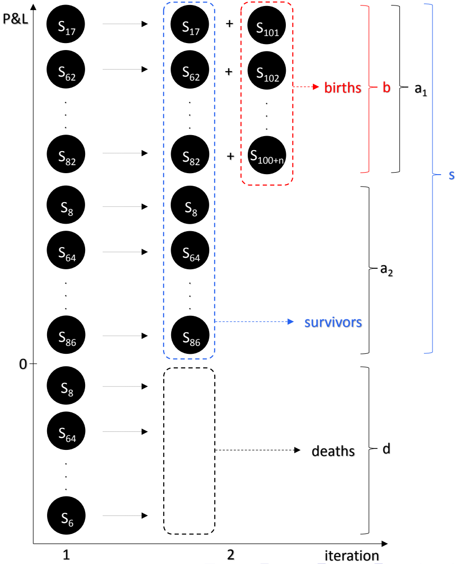
NN **depth & breadth** should contribute in **increasing the learning potential**. The TF (bottom) is less complex





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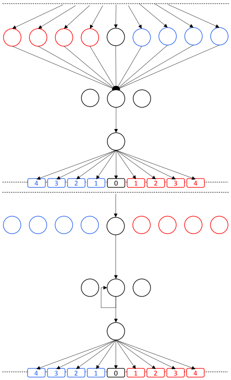
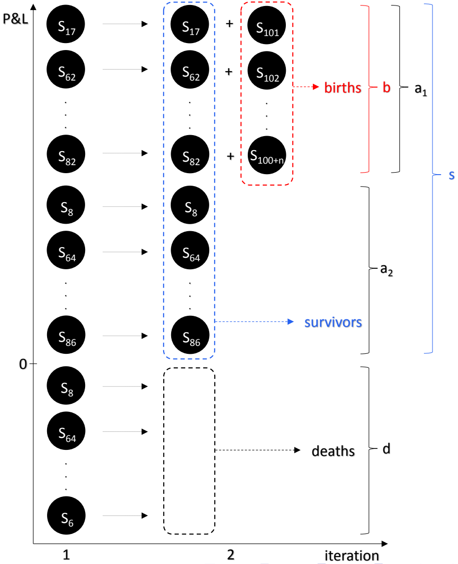
NN **depth & breadth** should contribute in **increasing the learning potential**. The TF (bottom) is less complex than the MLR (middle)





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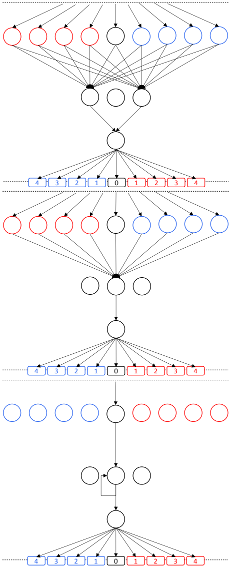

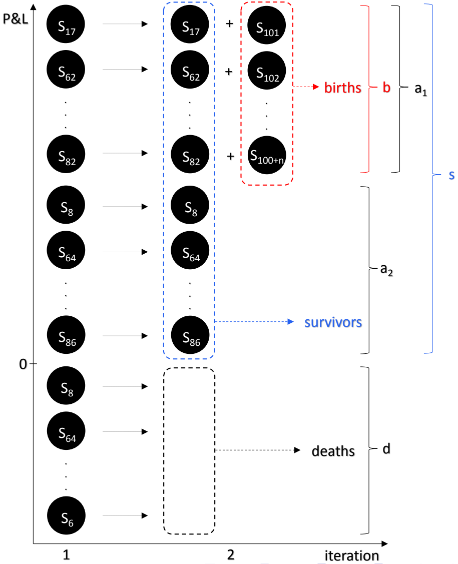

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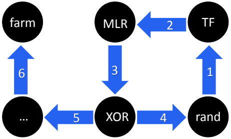
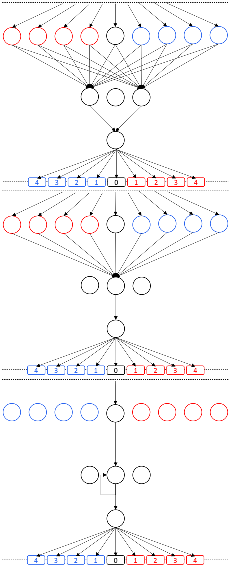
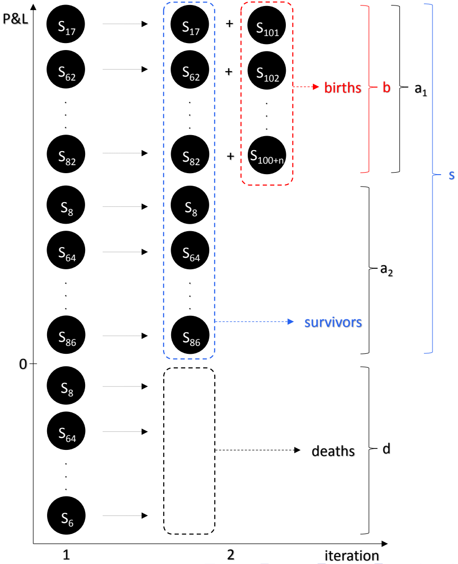
Question: Can we make an **analogy to the predator prey ecosystem**? Do we get similar behaviour as the Lotka-Volterra equations [28, 17]?







The Path of Interaction

Answer: not quite but there are **few interesting links** (exponential grow of the smaller prey/self fulfilling strategies such as TF) but they are **many issues** (classification, timescale etc...).

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Strategy	seed ⇈	TF1	TF2	TF1	TF2
Iteration	0		1		2
Signal	N/A	$+1$	$+1$	$+1$	$+1$
OB	${}_{0}\mathrm{P}^{1,1,1,1}$	${}_{0,0}\mathrm{P}^{1,1,1}$	${}_{0,0,0}\mathrm{P}^{1,1}$	${}_{0,0,0,0}\mathrm{P}^{1}$	${}_{0,0,0,0,0}\mathrm{P}$
Last Price	100	101	102	103	104
ΔOI	$+1$	-1	-2	-3	-4
ΔPrice	$+1$	$+1$	$+1$	$+1$	$+1$
P&L	$[0,0]$		$[1,0]$		$[2,1]$

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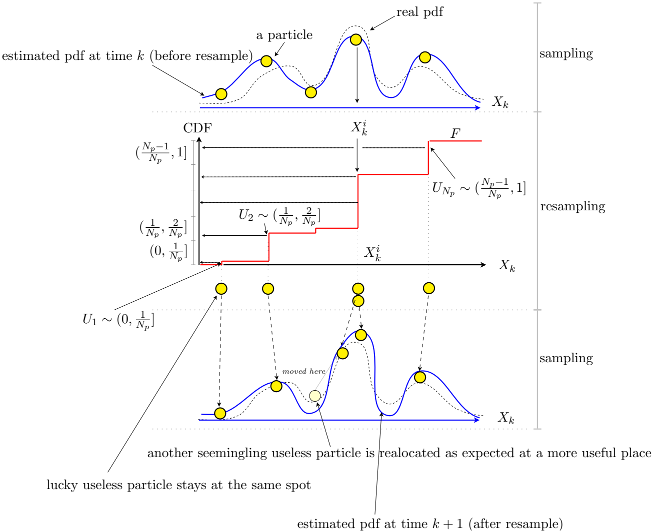
Sequential Monte Carlo

Sequential Monte Carlo (SMC) methods [9, 10, 16], also know as Particle Filter have emerged as a fashionable tool to **track scenarios** in the last 15 years [25, 26]. They are the sequential analogue of Markov Chain Monte Carlo (MCMC) methods and similar to importance sampling methods.



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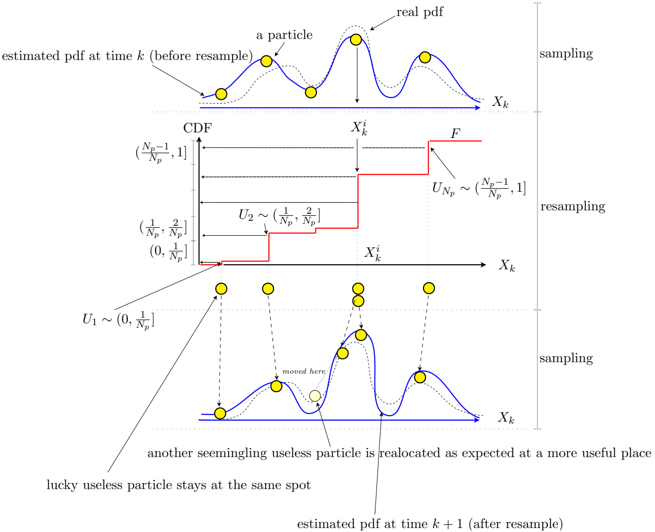
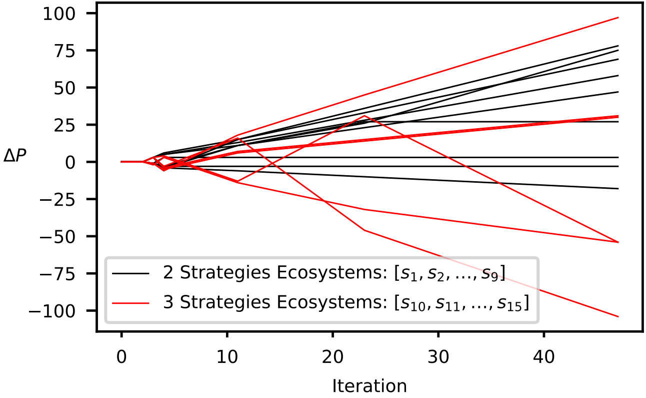
The aims of the PF is to **estimate the sequence of hidden parameters** (eg: the frequencies of certain types of strategies), **based on an indirect observations** (eg: the fluctuations of the market).



The random Brownian paths become deterministic



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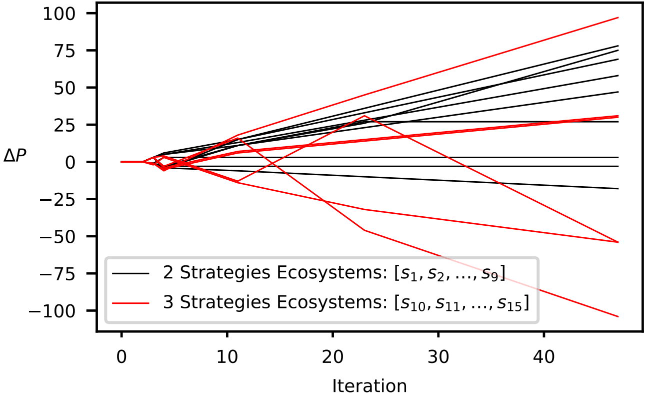
Each **Brownian Path Lookalike** is no longer stochastic but **deterministic**.



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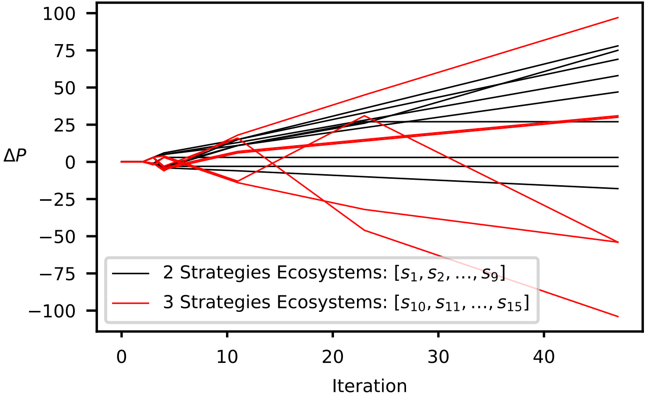

Each **deterministic path** corresponds to a **sequence of interaction** of several **strategies** for which the sequence and the **P&Ls** can be traced through our SMC methods by looking at the market **price only**.

PF assigns a probability for each ecosystem scenario

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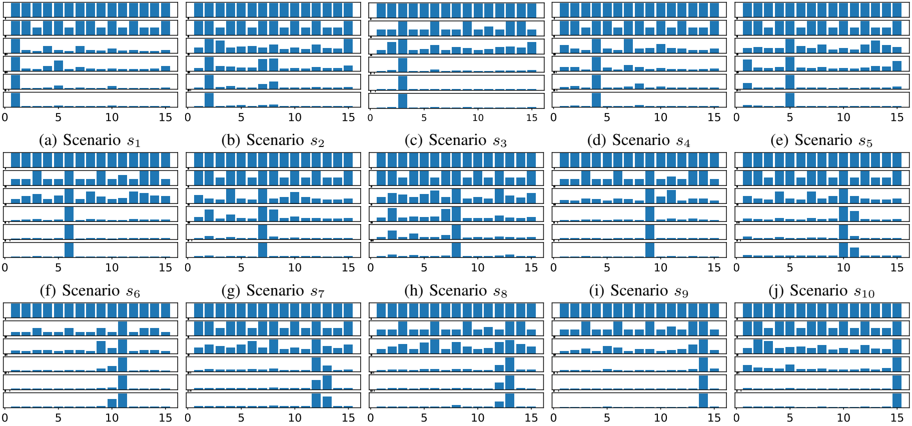

(a) Scenario s_1 (b) Scenario s_2 (c) Scenario s_3 (d) Scenario s_4 (e) Scenario s_5

(f) Scenario s_6 (g) Scenario s_7 (h) Scenario s_8 (i) Scenario s_9 (j) Scenario s_{10}

(k) Scenario s_{11} (l) Scenario s_{12} (m) Scenario s_{13} (n) Scenario s_{14} (o) Scenario s_{15}

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- Finally we looked at tracking methods using **MTT**.

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- **Birth & Death Processes**: We need to incorporate a Birth and Death Process to our MTT to make more realistic scenarios.
- **Diversity & Stability**: In biology diversity in an ecosystem leads to its instability [28, 7] but what about Finance?

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