Data-Driven Models & Mathematical Finance: Opposition or Apposition?

DPhil in Machine Learning Viva Voce

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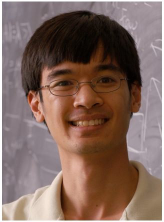
Successful Research Strategies



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- Some dedicate their whole lives to **one complicated problem** with a rare (but often nothing) outstanding outcome (Grigori Perelman).
- Some other prefer adhering to the "**publish or perish**" model at the cost of not producing the same quality research.
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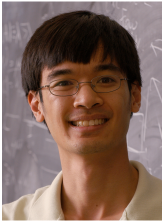

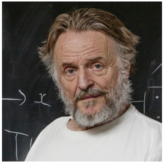

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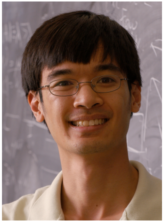
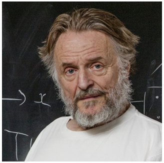
Theory vs Simulation

- **A good theory** should be able to be **simulated**.
- A good **simulation may change/iron out a theory**.
- **Cedric Villani** thinks that the process can go back and forth until the picture becomes clearer.





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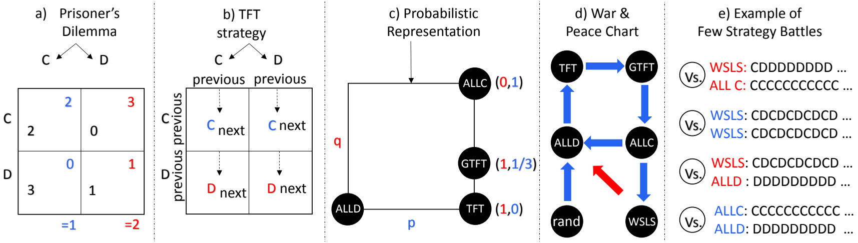

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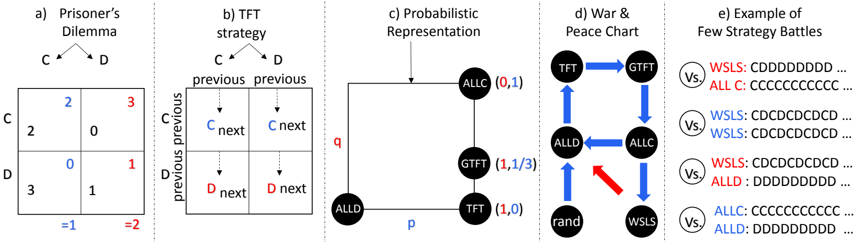

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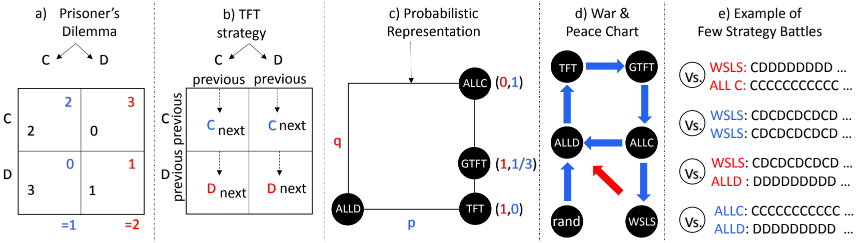

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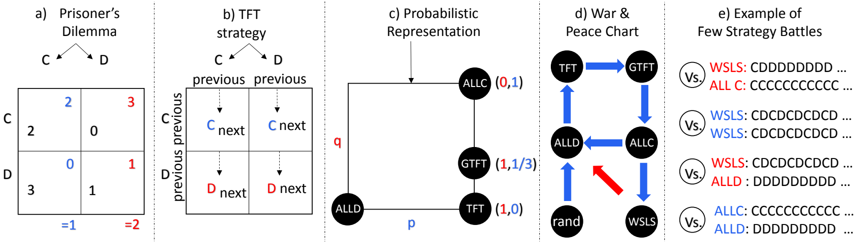

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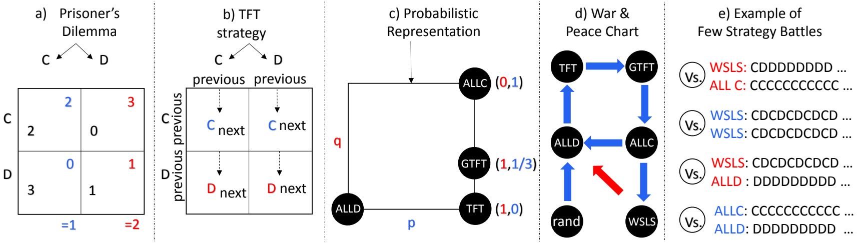

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The Rise of Big Data

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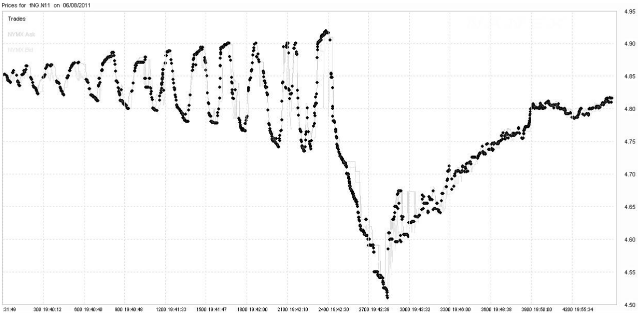

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- The **Flash Crashes** (eg: [29]) calls for a **modelling revolution** [5, 12, 6] (**BU vs. TD**): the Brownian motion assumption to model markets is increasingly difficult to defend.

1. Introduction
   - Foreword
   - Historical Context
   - Original Contribution
2. A Bottom-up Approach to the Financial Markets
   - Useful material for Quant Finance from Mathematical Biology
   - Neural Network Architecture and Learning Potential
   - Dynamic of the Financial Market
   - Stability of Financial Systems and Multi-Target Tracking
3. Model Assuming Data vs Data Reassuming the Models
   - Cointelation, Inferred Correlation & Portfolio Optimization
   - Anomaly Detection & Volatility Surface de-Arbitraging
   - Big Data Changing the Vanilla Options Landscape
   - Clustering for Distribution & Regime Change Forecasting
4. Conclusion
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- Formalizing the methodology for a **Particle Filter** which aim is to **track ecosystems of strategies** through time by looking at price dynamics only as well as performing few simulations.

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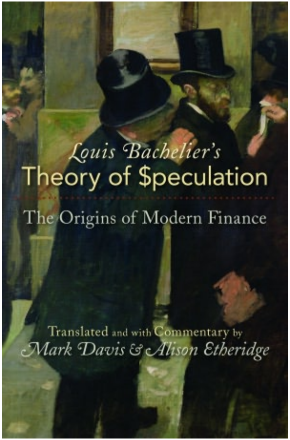
- Reconciling discordant Risk: **Anticipative Responsible** VaR.

Two simple questions for two simple definitions

Definition (Top-Down Vs Bottom-Up)

Top-Down (TD): Any **stochastic** quantitative approach which assumes that the **market is random** (or close to random) but for which we can create **interesting dynamic strategies**.



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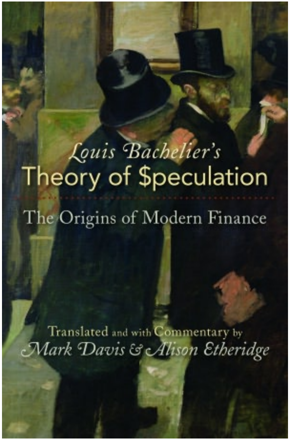

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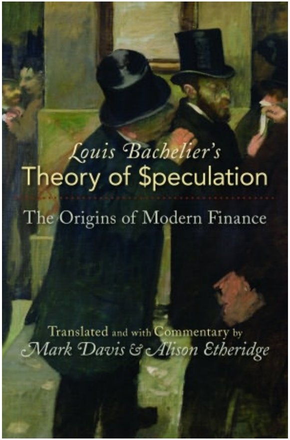

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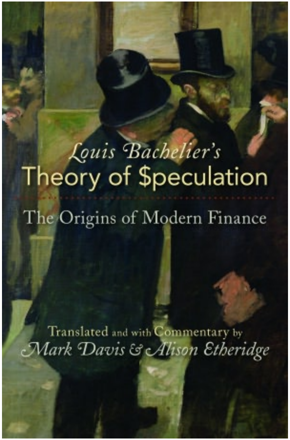

Can we use this new angle to **"solve"** the market?

The Scientific Method for "solving" the market

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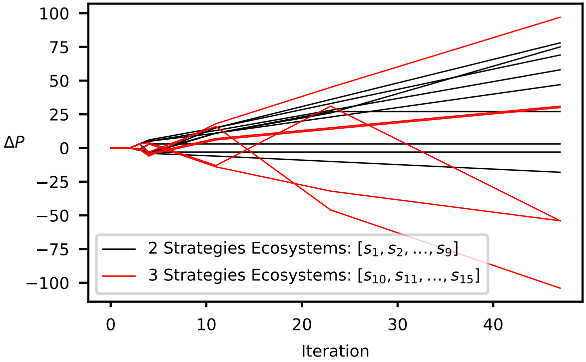

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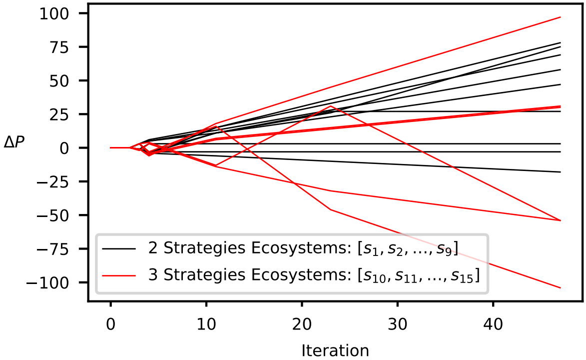

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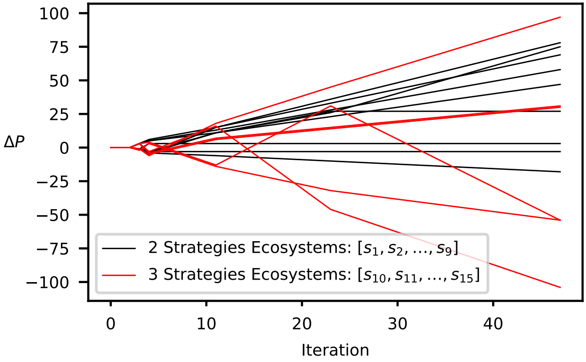
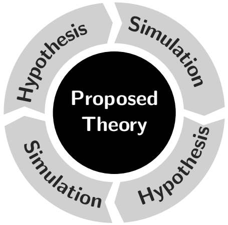


Scientific Method: A good theory can be simulated but simulations can also help bring intuition on what the theory might be [34].



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Predator/Prey models

biological ecosystems **predator/prey** (PP) [35, 8] models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between $x(t)$, $y(t)$ and $z(t)$ is **deterministic**:

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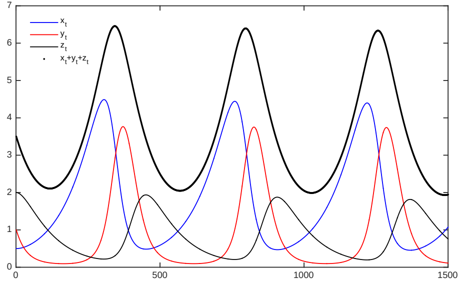

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We can make the **hypotheses that the economical cycles** or oscillation in prices are due to the same type of disruptions that can occur in biological ecosystems **predator/prey** (PP) [35, 8] models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between $x(t)$, $y(t)$ and $z(t)$ is **deterministic**:

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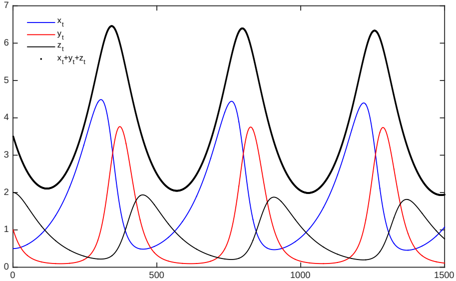

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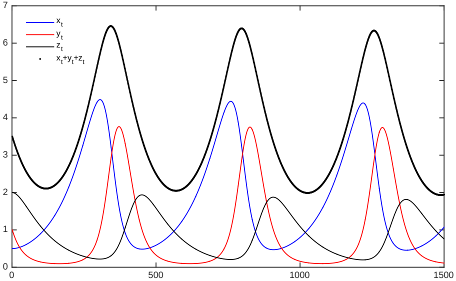
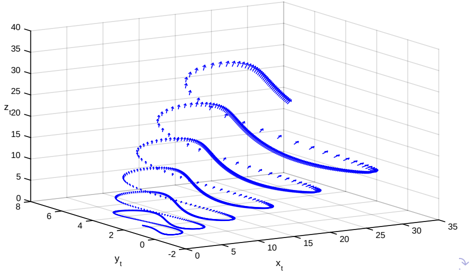


Answering if an **ecosystem** (or by extension financial market) composed of 3 strategies is **stable** would come to studying the Jacobian matrix J [8]. If if all **eigenvalues** of $J(x, y, z)$ have negative real parts then our system is asymptotically **stable**. Though simplistic, the model can easily be expanded to more **complex ecological niches**.

$$J(x, y, z) = \begin{bmatrix} a - by & -xb & 0 \\ yd & -c + dx - ez & -ye \\ 0 & -zg & -f + gy \end{bmatrix}$$



Evolutionary Dynamics

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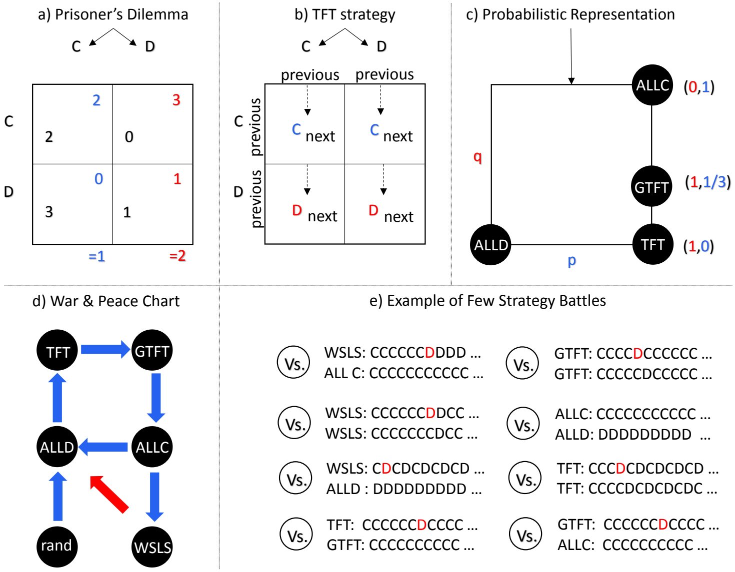

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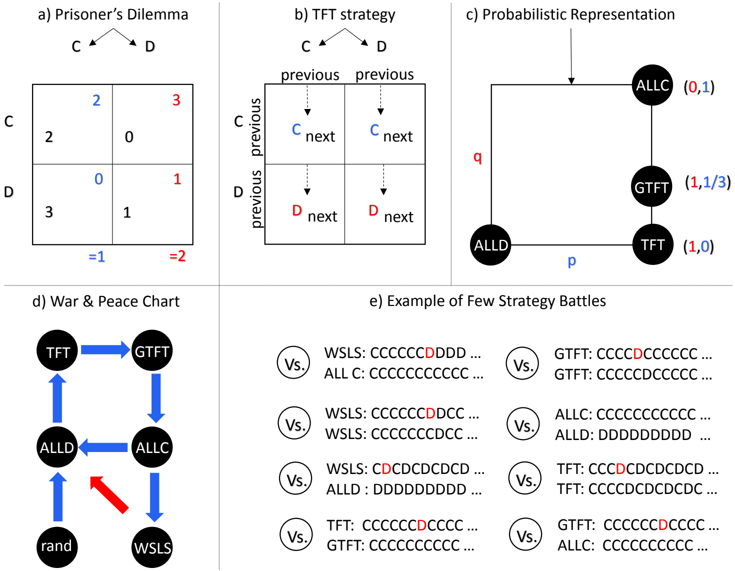

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The methodology in ED is interesting because the **strategies** are both **systematic & interacting** with each other (like it is the case in algo trading).

A first application in Economics: Minority Game

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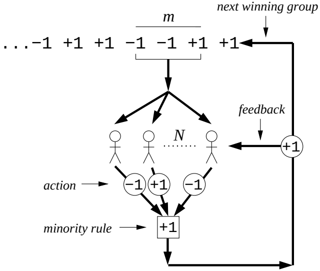
input	output
−1 −1 −1	−1
−1 −1 +1	−1
−1 +1 −1	+1
−1 +1 +1	−1
+1 −1 −1	−1
+1 −1 +1	+1
+1 +1 −1	−1
+1 +1 +1	+1



A first application in Economics: Minority Game

In the **Minority Game** [7], developped by Challet, Marsili and Zhang, players need to choose between two options $(+1, -1)$. Those who have selected the option chosen by the minority "win".

input	output
−1 −1 −1	−1
−1 −1 +1	−1
−1 +1 −1	+1
−1 +1 +1	−1
+1 −1 −1	−1
+1 −1 +1	+1
+1 +1 −1	−1
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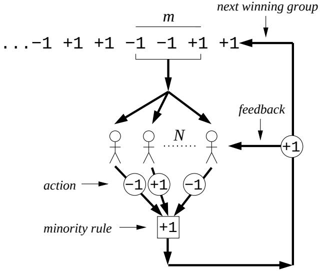

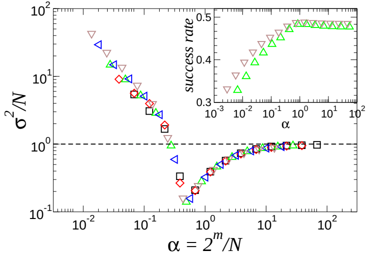

Physical Laws can emerge from these simple rules. We can observe that σ^2/N is only a function of $\alpha = 2m/N$ which considering the complexity of the interactions between the set of agents can be quite remarkable.

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+1 −1 −1	−1
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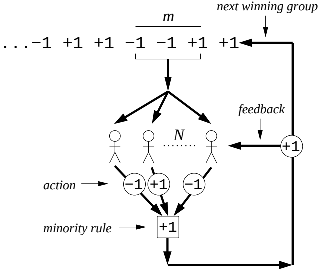

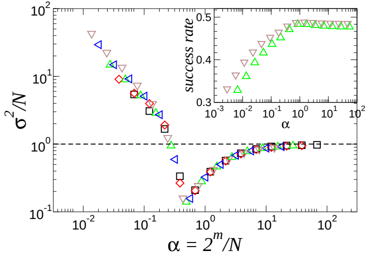

Criticism: Is this realistic for Economics? Maybe, but not for algorithmic trading (eg: TF strategy in a TF concentrated ecosystem)?

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Bringing these Ideas into Electronic Trading

- As ML "translation" of ED and PP models, we have **Generative Adversarial Networks** (GANs) [14], introduced in 2014, usually **involve** a system of **two neural networks** competing in a zero-sum game settings. This process continues as long as needed since the lack of **data is no longer a problem**.

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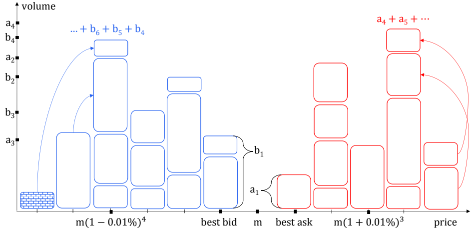

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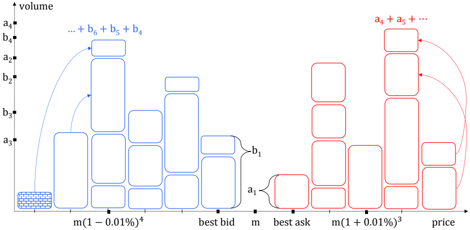
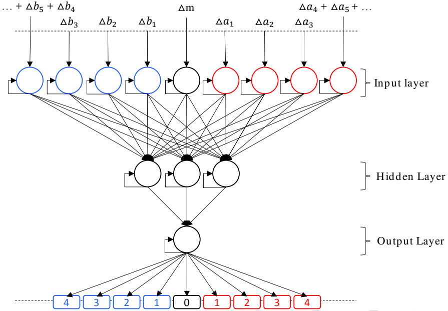
- The **communication tool** is the **order book** and the game is **not zero-sum game** in our research. The strategies are in **High Frequency Financial Funnel** (HFFF) format [23].

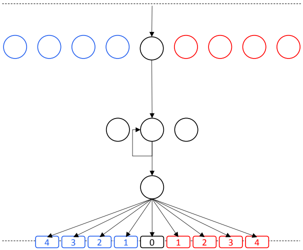




High Frequency Financial Funnel & Classic Strategies

HFFF can model financial strategies:

High Frequency Financial Funnel & Classic Strategies



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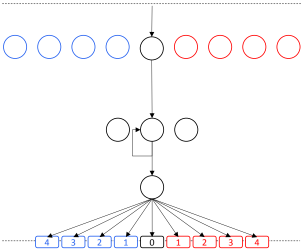
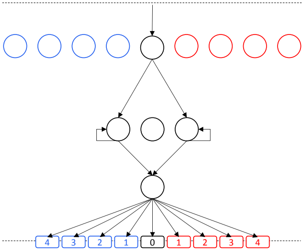
1. Trend Following (TF)

High Frequency Financial Funnel & Classic Strategies





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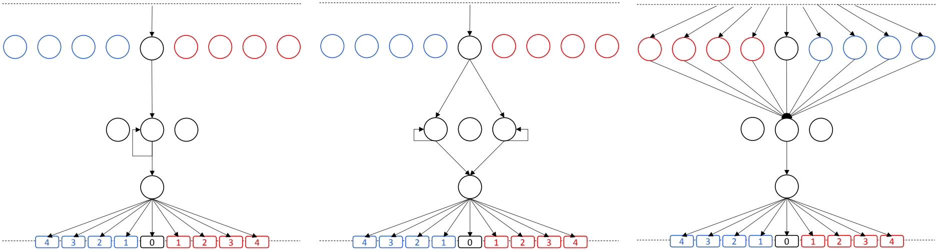
High Frequency Financial Funnel & Classic Strategies



HFFF can model financial strategies:

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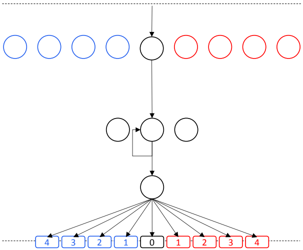
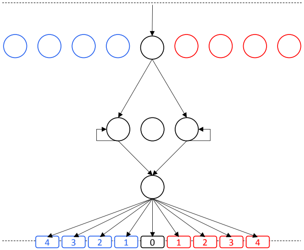
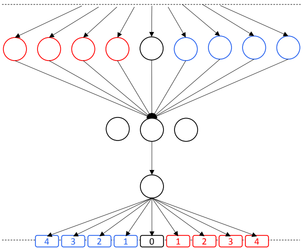
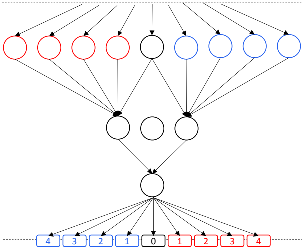
3. MLR









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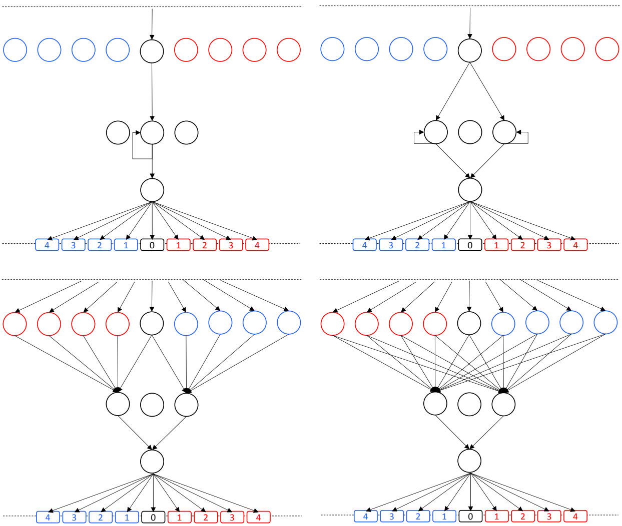
High Frequency Financial Funnel & Classic Strategies



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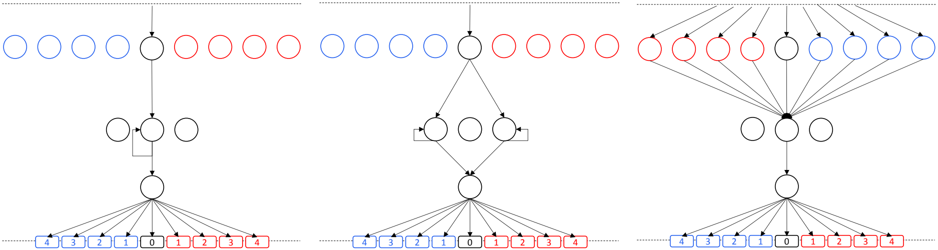

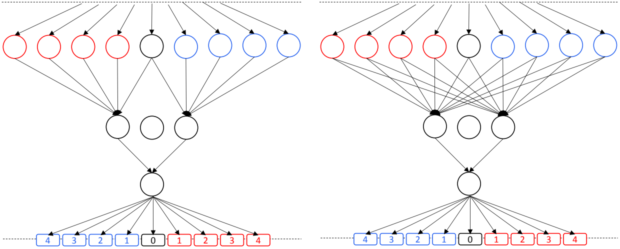

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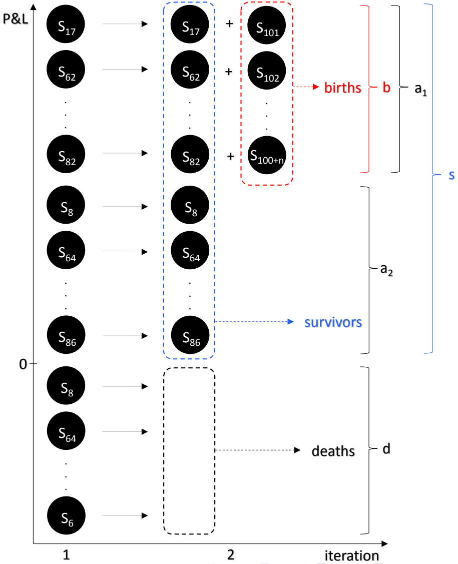
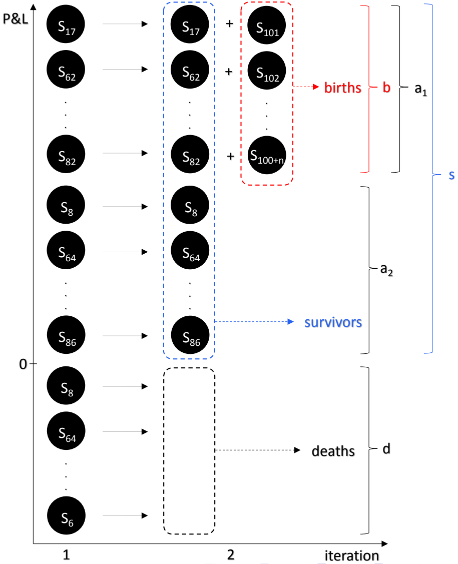
Architecture Complexity and **strategy sophistication** explains the **incentive for Deep Learning** (DL).
Paradoxically we witness **potential for regularization** as the network becomes more complex.

Genetic Algorithm & NN Complexity



Genetic Algorithm & NN Complexity

NN **depth & breadth** should contribute in **increasing the learning potential**.



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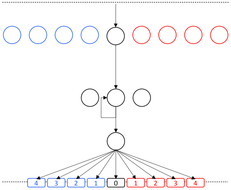
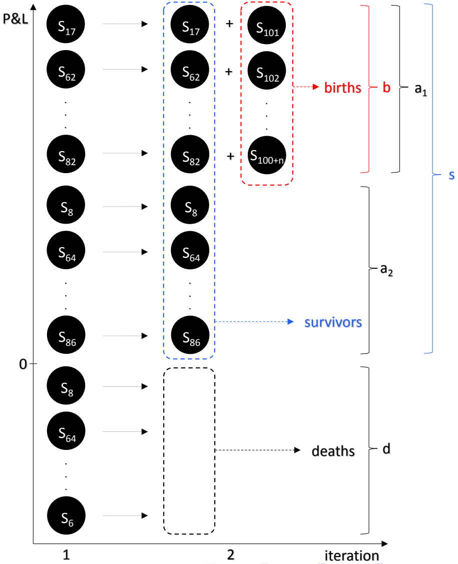
NN **depth & breadth** should contribute in **increasing the learning potential**. The TF (bottom) is less complex





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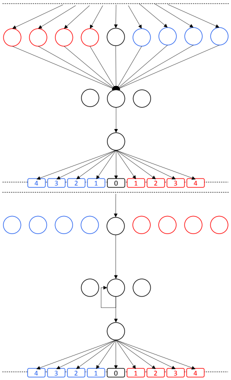

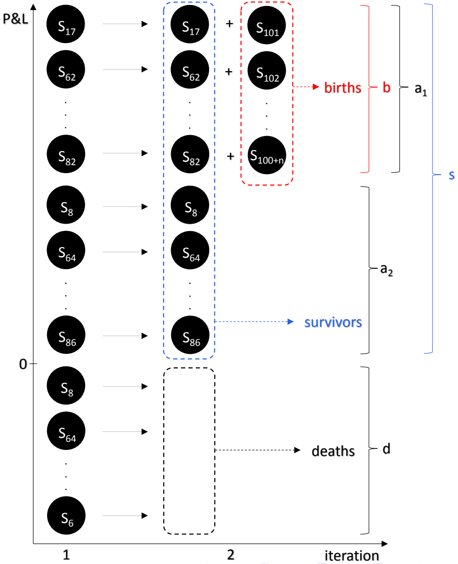

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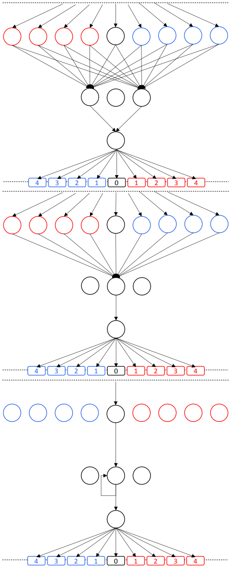

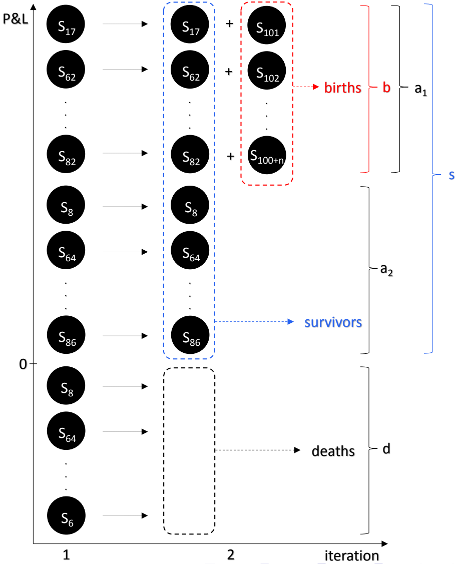

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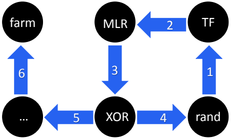
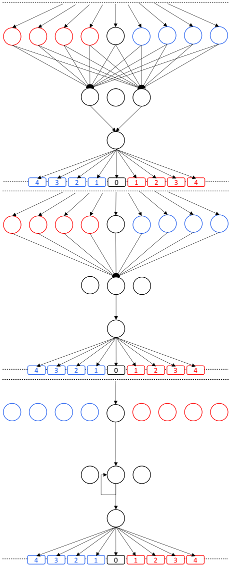
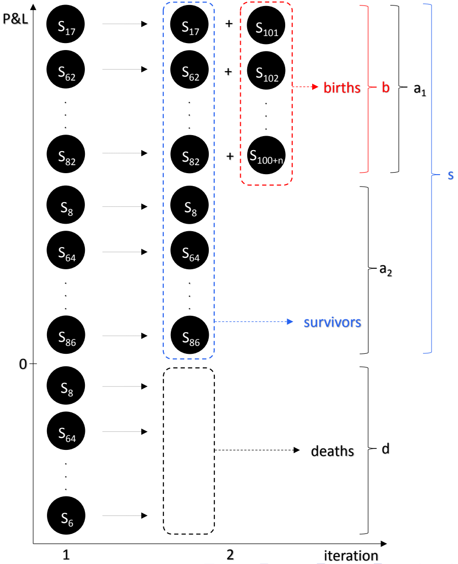
Question: Can we make an **analogy to the predator prey ecosystem**? Do we get similar behaviour as the Lotka-Volterra equations [35, 19]?







The Path of Interaction

Answer: not quite but there are **few interesting links** (exponential grow of the smaller prey/self fulfilling strategies such as TF) but they are **many issues** (classification, timescale etc...).

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Strategy	seed ⇈	TF1	TF2	TF1	TF2
Iteration	0		1		2
Signal	N/A	$+1$	$+1$	$+1$	$+1$
OB	${}_{0}\mathrm{P}^{1,1,1,1}$	${}_{0,0}\mathrm{P}^{1,1,1}$	${}_{0,0,0}\mathrm{P}^{1,1}$	${}_{0,0,0,0}\mathrm{P}^{1}$	${}_{0,0,0,0,0}\mathrm{P}$
Last Price	100	101	102	103	104
ΔOI	$+1$	-1	-2	-3	-4
ΔPrice	$+1$	$+1$	$+1$	$+1$	$+1$
P&L	$[0,0]$		$[1,0]$		$[2,1]$

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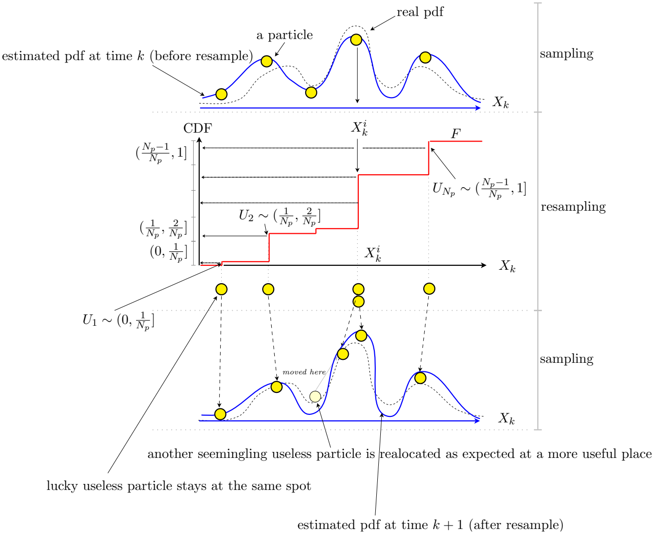
Sequential Monte Carlo

Sequential Monte Carlo (SMC) methods [10, 11, 18], also know as Particle Filter have emerged as a fashionable tool to **track scenarios** in the last 15 years [31, 32]. They are the sequential analogue of Markov Chain Monte Carlo (MCMC) methods and similar to importance sampling methods.



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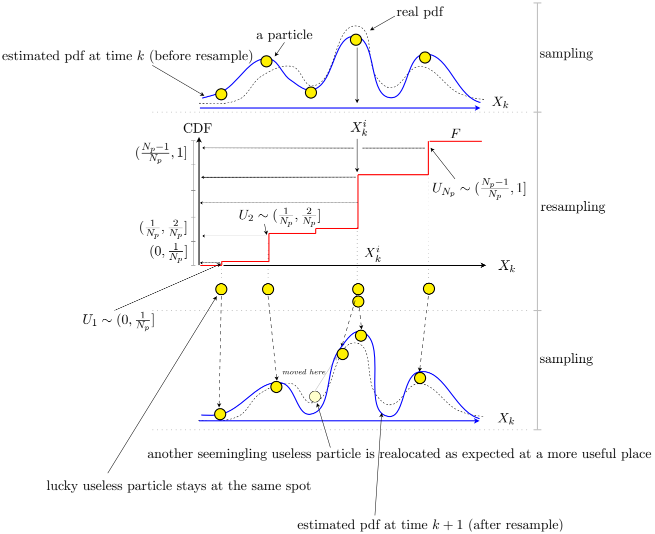
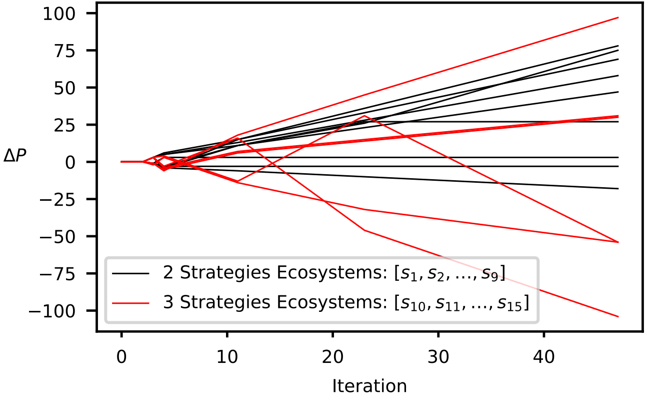
The aims of the PF is to **estimate the sequence of hidden parameters** (eg: the frequencies of certain types of strategies), **based on an indirect observations** (eg: the fluctuations of the market).



The random Brownian paths become deterministic



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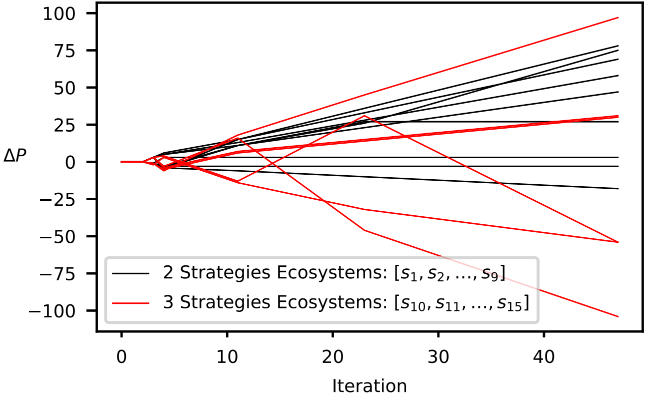
Each **Brownian Path Lookalike** is no longer stochastic but **deterministic**.



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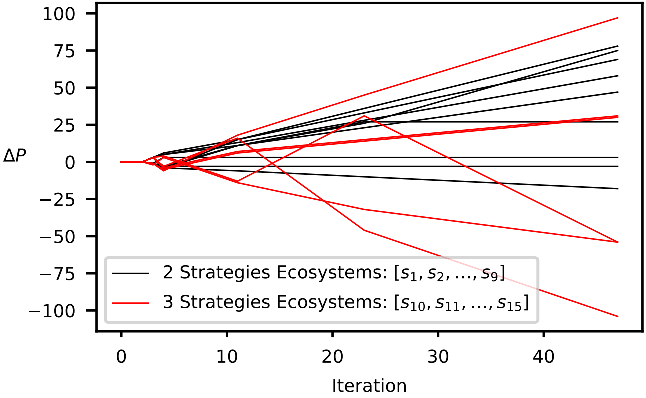

Each **deterministic path** corresponds to a **sequence of interaction** of several **strategies** for which the sequence and the **P&Ls** can be traced through our SMC methods by looking at the market **price only**.

PF assigns a probability for each ecosystem scenario

We have recorded **15 different scenarios** (ecosystem history) for the sake of this presentation, all of which are clearly detected after the 11th iteration [27].

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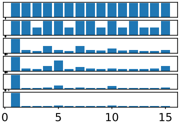
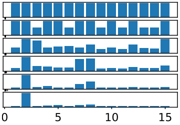
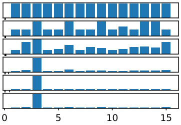


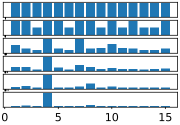
(a) Scenario s_1



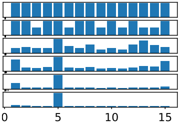
(b) Scenario s_2



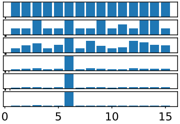
(c) Scenario s_3



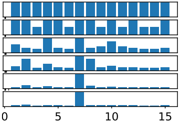
(d) Scenario s_4



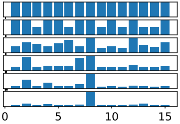
(e) Scenario s_5



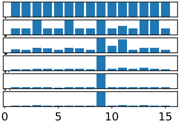
(f) Scenario s_6



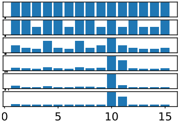
(g) Scenario s_7



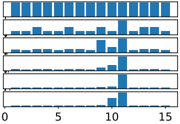
(h) Scenario s_8



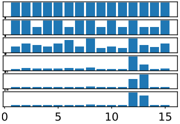
(i) Scenario s_9



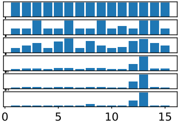
(j) Scenario s_{10}



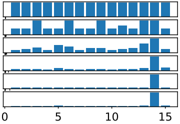
(k) Scenario s_{11}



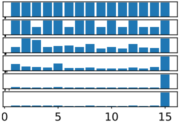
(l) Scenario s_{12}



(m) Scenario s_{13}



(n) Scenario s_{14}



(o) Scenario s_{15}

Model Assuming Data

Definition (Correlation Model)

On a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{(t\geq 0)}, \mathbb{P})$ the Correlation model is given by the set of two SDE's:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t),$$

$$dY(t) = \mu Y(t)dt + \sigma Y(t)d\tilde{W}(t), \tag{1}$$

$$d\langle W, \tilde{W}\rangle_t = \rho dt, \tag{2}$$

where $\mu \in \mathbb{R}$, $\sigma > 0$ are drift and diffusion coefficients of asset price, $\tilde{W}(t)$ and $W(t)$ are two correlated Brownian motions with constant correlation coefficient $\rho \in [-1, 1]$.

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Remark: few **assumptions** need to be fulfilled. Variance must remain constant (**homoscedasticity**) and the **returns independent**. If not instantaneous measured correlation is misleading with respect to long term correlation:

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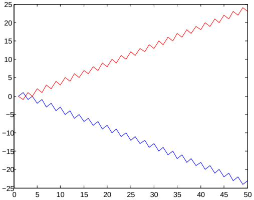

Remark: few **assumptions** need to be fulfilled. Variance must remain constant (**homoscedasticity**) and the **returns independent**. If not instantaneous measured correlation is misleading with respect to long term correlation:

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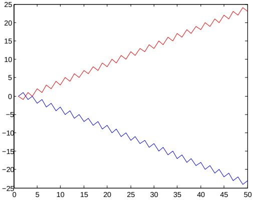

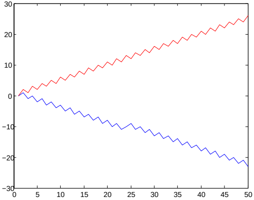

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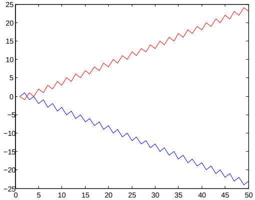

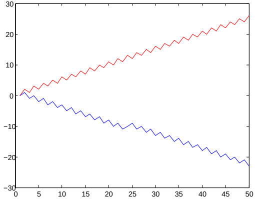

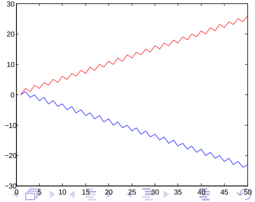

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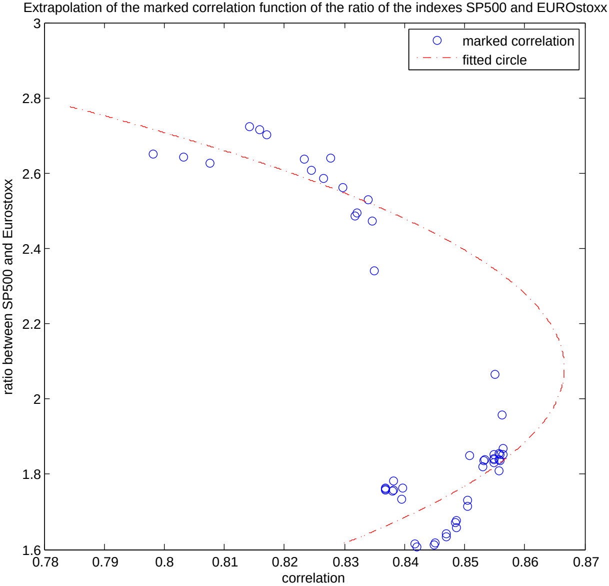
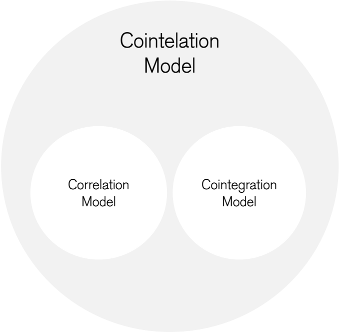
- or **indirectly** (Figure on left hand side). Implied correlation is "marked", the same way implied volatility is. The ratio of two closely related underliers exhibits mean reversion in the minds of the traders risk managing these products.



Data Reassuming the Model

Intuitive Definition: Cointelation is a portmanteau neologism in finance, designed to signify a hybrid method between between **cointe**gration and corre**lation** models (Data-Driven adjustment to classic financial math models: data "reassuming the model").



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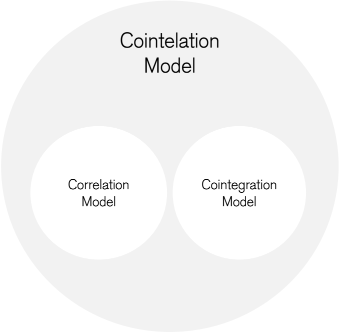

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Remark: We note that setting $\kappa = 0$ yields the classic correlation model. Conversely, setting $\rho = 0$ yields the cointegration model.

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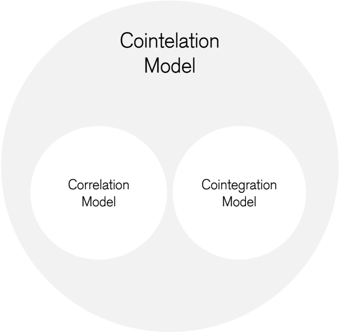
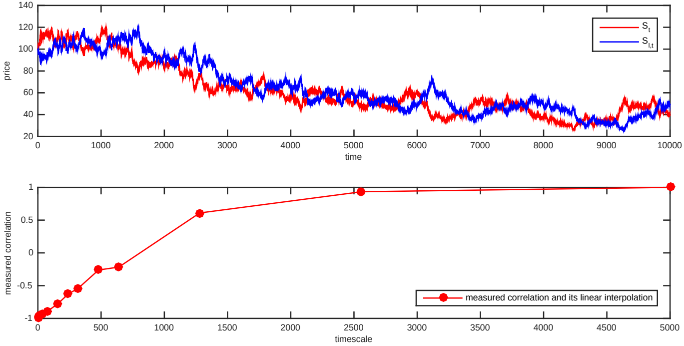


Interesting Property: the Inferred Correlation



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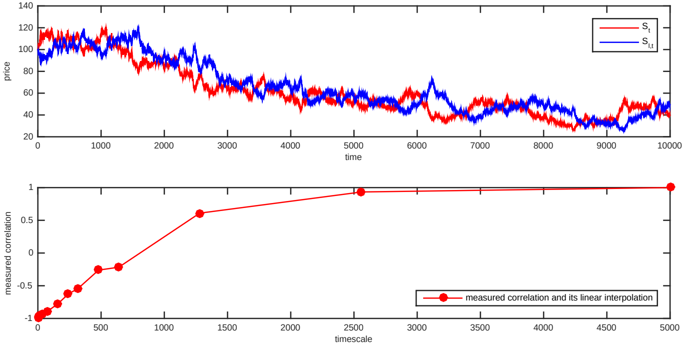

Remark: Cointelation model **can hit the whole correlation spectrum** depending on timescale.

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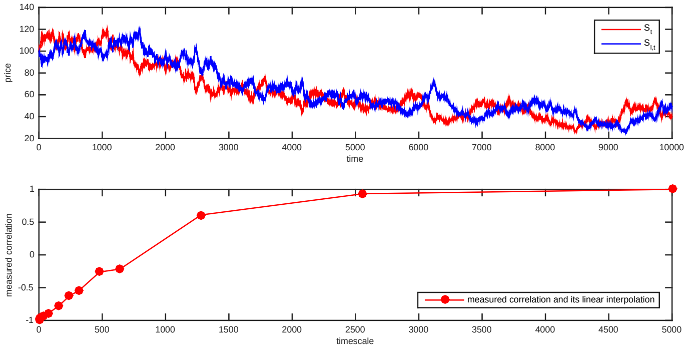
Definition (Inferred Correlation)

Considering, the dynamics of equation (3), the **Inferred Correlation** formula is given by (3).

$$\rho^*_\tau \approx \rho + (1-\rho)\left[1-\exp\left(-\kappa\lambda(\tau-1)\right)\right] \quad (3)$$

where $\rho^*_\tau = \mathbb{E}[\sup_{0<t\leq\tau} \rho_t]$, $\tau \in \mathbb{Z}^*$, $\theta \in [0,1]$ and λ constant.



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Application: Socially Responsible Finance

Socially Responsible & Consumer Finance is a wave of quant finance that gains momentum after each crisis but quickly runs unfortunately out of fuel:

- **Situation**: Equity/Commodities salesman trying to convince clients who have a portfolio in the Commodities/Equities asset class to diversify by buying salesman's products.
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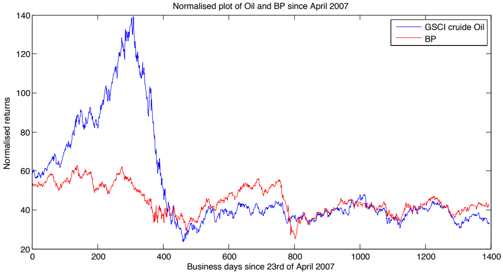

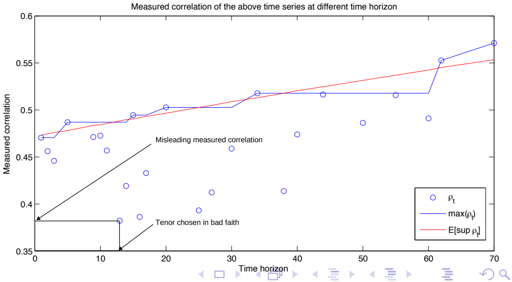

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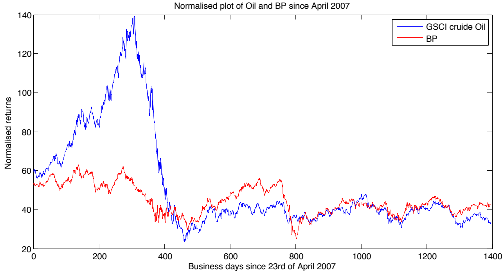

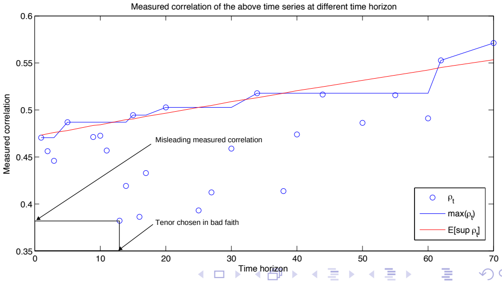

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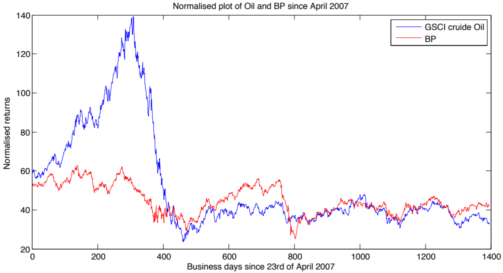

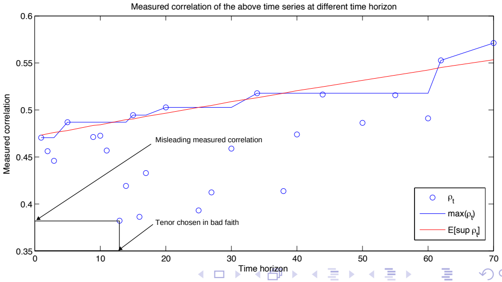

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The **red part** does not have a closed for solution: **the pure Classic Financial Mathematics does not have a solution**.

Portfolio Optimization: ML/FM Hybrid Method

The way to handle the **red part** part is to consider the function $G(t, v, s)$ such that $G \in C^{1,2}(Q)$, the Hamilton-Jacobi-Bellman (**HJB**) equation corresponding to stochastic control problem is

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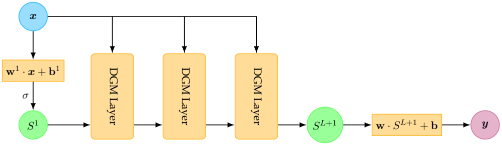

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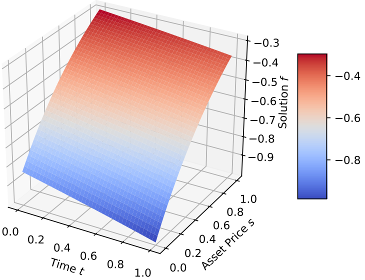

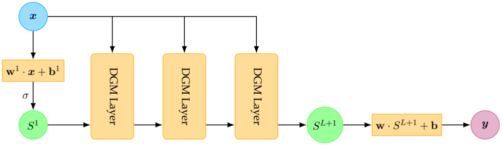

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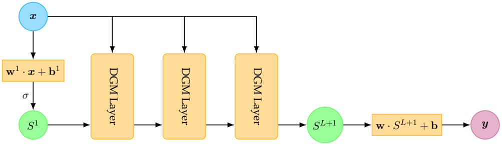

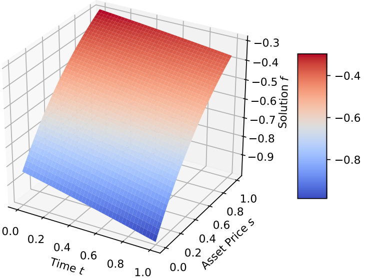

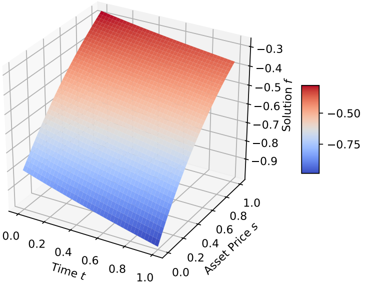

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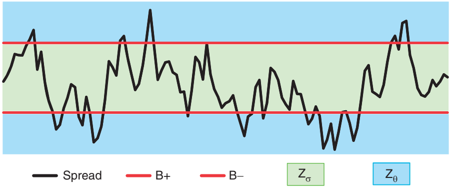

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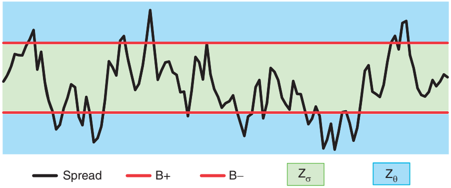

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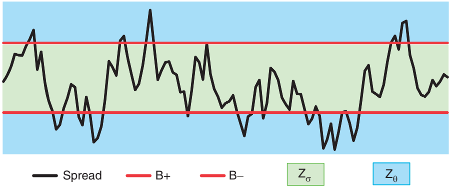

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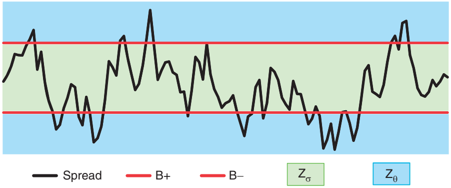

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- Strategy S^{-+} in which we are short X and long Y at time t within $[a_i, b_i]$, $i \in \mathbb{N}$, and with P&L $V^{-+}_{[a_i,b_i],t}$.
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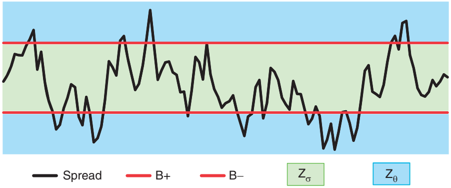

Portfolio Optimization: Pure ML Approximation

We can use clustering in order to study the P&L of n (eg: 4) strategies through time.

- Strategy S^{++} in which we are long both X and Y at time t within bands $[a_i, b_i]$, $i \in \mathbb{N}$, and with P&L $V^{++}_{[a_i,b_i],t}$.
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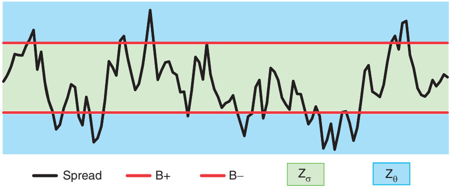
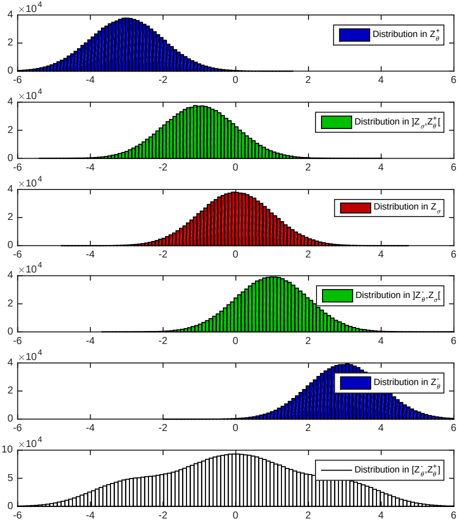
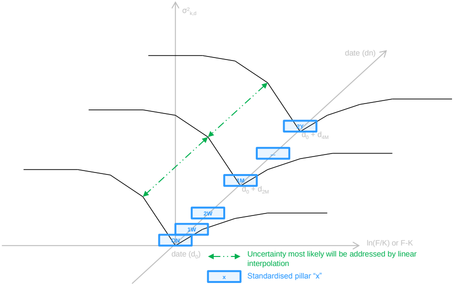




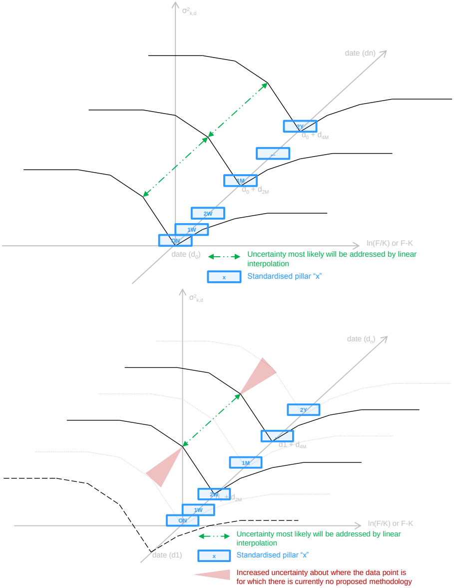
Remark: We can have as many bands (strategies) as we have weight proportions.

Anomaly Detection & Volatility Surface de-Arbitraging



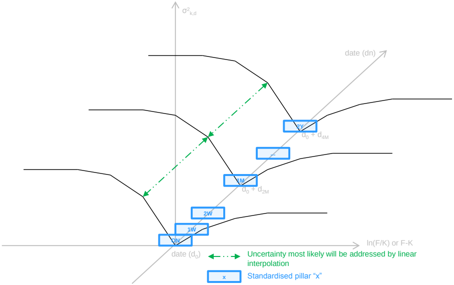
Normalizing the data coming from the markets (left figure)

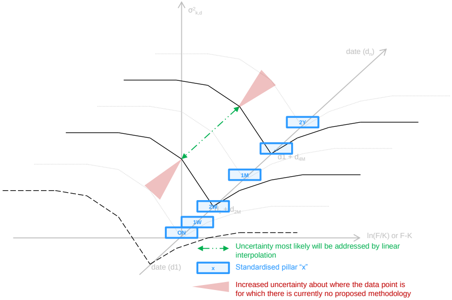
Anomaly Detection & Volatility Surface de-Arbitraging



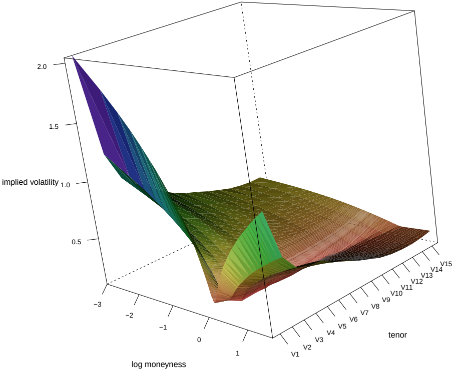
Normalizing the data coming from the markets (left figure) in **rolling contract form**

Anomaly Detection & Volatility Surface de-Arbitraging





Normalizing the data coming from the markets (left figure) in **rolling contract form** into a coherent **fixed pillars implied volatility surface** (bottom right figure) presents challenges.



Anomaly Detection & Volatility Surface de-Arbitraging

The classic arbitrage conditions (butterfly & calendar spread) have been replaced with more elegant models (e.g. $\forall K, \forall T, |T\partial_K \sigma^2(K,T)| \leq 4$) that were nevertheless not sufficient and the arrival of Big Data in the wings exposed these limitations in the Financial Mathematics models.

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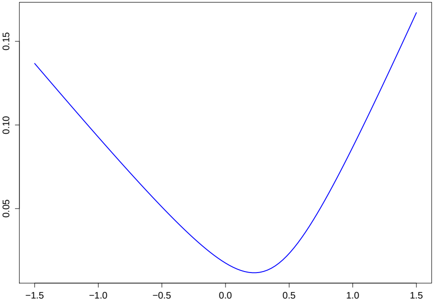


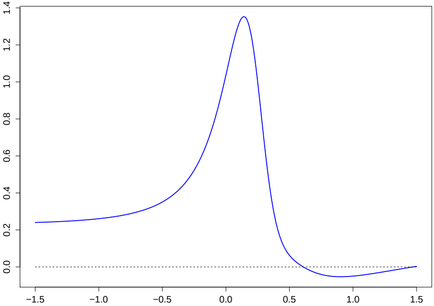
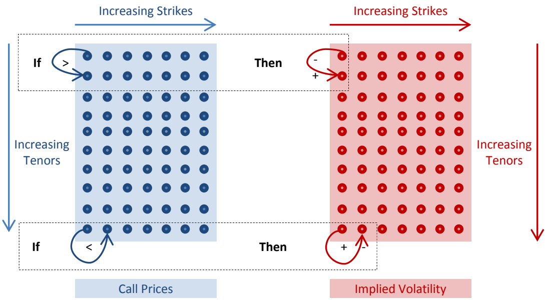


Figure: Vogt's total variance example verifying $b(1+|\rho|) \leq \frac{4}{T}$ (left figure: with the x axis being the log-moneyness and the y axis being the implied variance) and the corresponding $\partial^2_{K,K} BS\left(\sigma^2(K,T)\right)$ approximating the (supposed) always positive pdf (right figure: with the x axis being the log-moneyness and the y axis being the non normalized pdf).

Anomaly Detection & Volatility Surface de-Arbitraging

The work of this chapter revolves around **anomaly detection** in the context of the implied volatility surface trying to use the **classic methods**: butterfly arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the **more advanced methods** such as in equation (5e) (a **necessary but not sufficient** condition).

Anomaly Detection & Volatility Surface de-Arbitraging



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$$\text{solve: } \hat{\sigma}_t(\tau, d) = \underbrace{\arg\min}_{\tilde{\sigma}_t(\tau,d)} \sum_{\tau} \sum_{d} \left[C\Big(\sigma_{i,t}(\tau, d)\Big) - C\Big(\tilde{\sigma}_t(\tau, d)\Big)\right]^2 \quad (5a)$$

subject to: $\forall \tau$ and $\forall K$

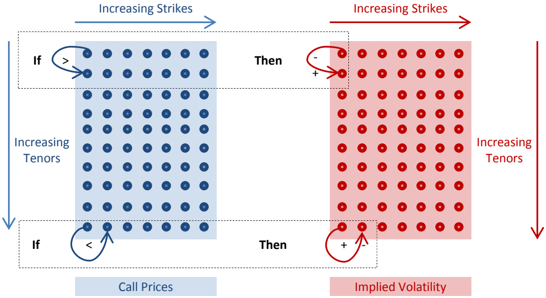
$$C\Big(K - \Delta, \sigma_0(K - \Delta, \tau)\Big) - C\Big(K, \sigma_0(K, \tau)\Big) \geq 0 \quad (5b)$$

$$C\Big(K, \tau + \Delta, \sigma_0(K, \tau + \Delta)\Big) - C\Big(Ke^{-r\Delta}, \sigma_0(Ke^{-r\Delta}, \tau)\Big) \geq 0 \quad (5c)$$

but not: (5d)

$$\forall K, \forall T, |T \partial_K \sigma^2(K, T)| \leq 4 \quad (5e)$$

Anomaly Detection & Volatility Surface de-Arbitraging



The work of this chapter revolves around **anomaly detection** in the context of the implied volatility surface trying to use the **classic methods**: butterfly arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the **more advanced methods** such as in equation (5e) (a **necessary but not sufficient** condition). We also discuss some of the **idiosyncratic asset class related differences** that require modifications in the optimization process. Finally we discuss few **trading idea by simplifying the IVP** model.

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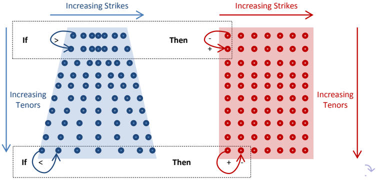
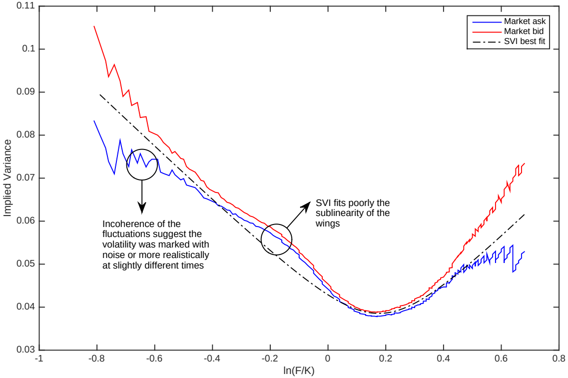


Big Data Changing the Vanilla Options Landscape



- exposed the limitations of both the **wings** (e.g. **SVI**) and of the **liquidity** in the options market,

We saw in the **Options market**:

- a step back in **complexity** (from exotics to vanilla) but with more **coherent** pricing
- a step forward in **liquidity** modelling

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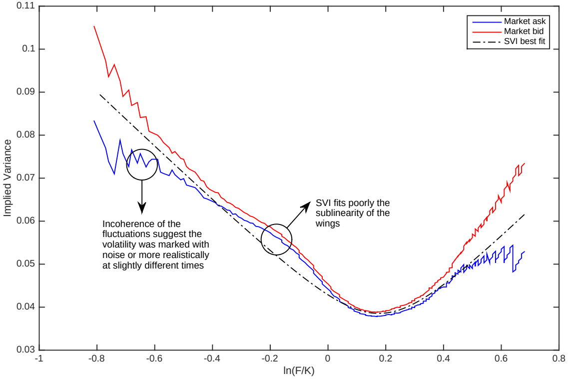

- exposed the limitations of both the **wings** (e.g. **SVI**) and of the **liquidity** in the options market,
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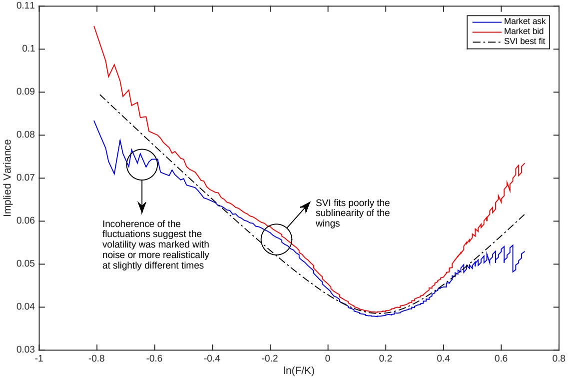

$$dS_t = \sqrt{v_t} S_t dW_t^1, \quad S_0 \in \mathbb{R}_+^* \tag{6a}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2, \quad v_0 \in \mathbb{R}_+^* \tag{6b}$$

The **rise of big data**:

- exposed the limitations of both the **wings** (e.g. **SVI**) and of the **liquidity** in the options market,
- the need for **proxying** when the data is scarce.
- the **Heston** (Stochastic Volatility) model and the local volatility model and the need for **harmonizing** these two concepts.

We saw in the **Options market**:

- a step back in **complexity** (from exotics to vanilla) but with more **coherent** pricing
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Big Data Changing the Vanilla Options Landscape

Big Data exposed the limitations of the **SVI** in the wings and the **subprime crisis** of 2007 exposed the need to incorporate **liquidity** directly in the options model.

Big Data Changing the Vanilla Options Landscape

We define the Implied Volatility surface Parametrization (IVP) split with its mid in equation (7) with the **downside transform** in equation (7b) **enhancing the SVI**,

$$\sigma^2_{IVP,o,\tau}(k) = a_\tau + b_\tau \left[\rho_\tau \left(z_{o,\tau} - m_\tau \right) + \sqrt{\left(z_{o,\tau} - m_\tau \right)^2 + \sigma^2_\tau} \right] \quad (7a)$$

$$z_{o,\tau} = \frac{k}{\beta_{o,\tau}^{1+4|k-m|}} \quad (7b)$$

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and its liquidity parameters in equation (8). Parameters ψ represents the **Wings Curvature**, α represents the **ATM Spread**. The latter two parameters can be defined in terms of functions to **accommodate the position size** in which case the market dept is controlled with the η parameters.

$$\sigma^2_{IVP,+,\tau}(k) = a_\tau + b_\tau \left[\rho_\tau \left(z_{+,\tau} - m_\tau \right) + \sqrt{\left(z_{+,\tau} - m_\tau \right)^2 + \sigma^2_\tau} \right] + \alpha_\tau(p) \quad (8a)$$

$$z_{+,\tau} = z_{o,\tau}[1 + \psi_\tau(p)] \quad (8b)$$

$$\sigma^2_{IVP,-,\tau}(k) = a_\tau + b_\tau \left[\rho_\tau \left(z_{-,\tau} - m_\tau \right) + \sqrt{\left(z_{-,\tau} - m_\tau \right)^2 + \sigma^2_\tau} \right] - \alpha_\tau(p) \quad (8c)$$

$$z_{-,\tau} = z_{o,\tau}[1 - \psi_\tau(p)] \quad (8d)$$

$$\alpha_\tau(p) = \alpha_{0,\tau} + (a_\tau - \alpha_{0,\tau})(1 - e^{-\eta_{\alpha_\tau} p}) \quad (8e)$$

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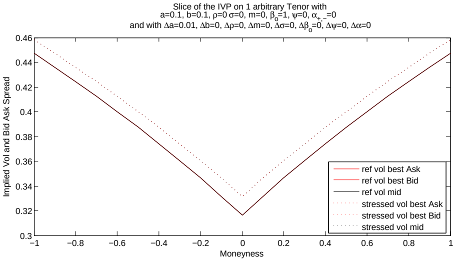
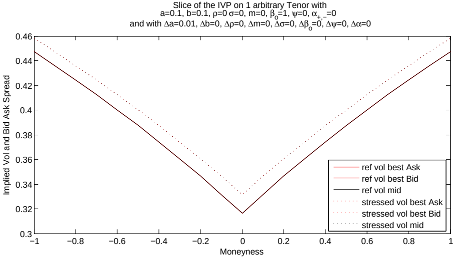
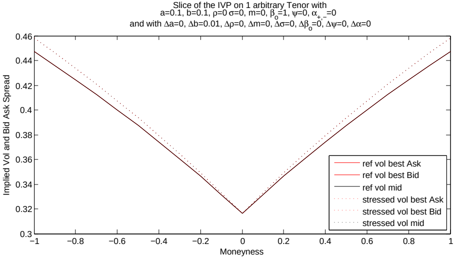
Big Data exposed the limitations of the **SVI** in the wings and the **subprime crisis** of 2007 exposed the need to incorporate **liquidity** directly in the options model. The **IVP** addresses these two points as well as allow for additional benefits:

- **Proxying**
- **Backtesting** weaponry on complex strategies.


Slice of the IVP on 1 arbitrary Tenor with
a=0.1, b=0.1, ρ=0 σ=0, m=0, β_o=1, ψ=0, α_{+,−}=0
and with Δa=0.01, Δb=0, Δρ=0, Δm=0, Δσ=0, Δβ_o=0, Δψ=0, Δα=0
Implied Vol and Bid Ask Spread
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Moneyness
ref vol best Ask
ref vol best Bid
ref vol mid
stressed vol best Ask
stressed vol best Bid
stressed vol mid

Big Data, Proxying & Handling Dimensionality for Options





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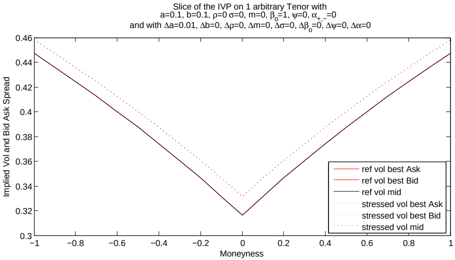

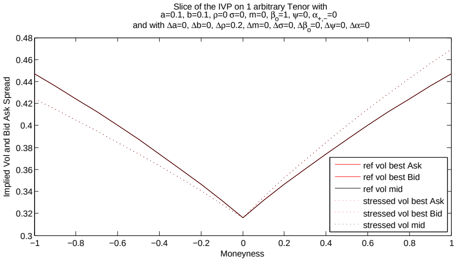

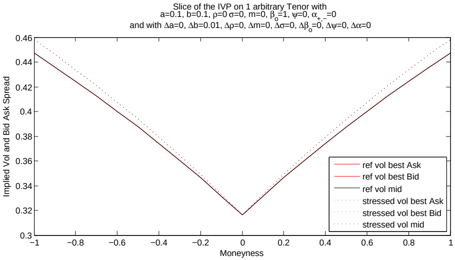

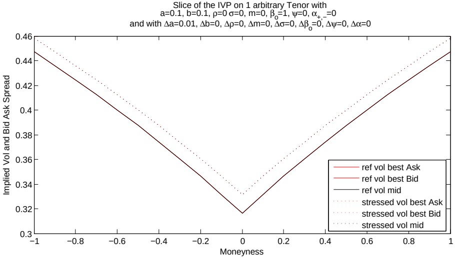
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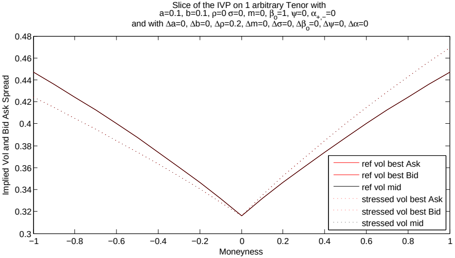
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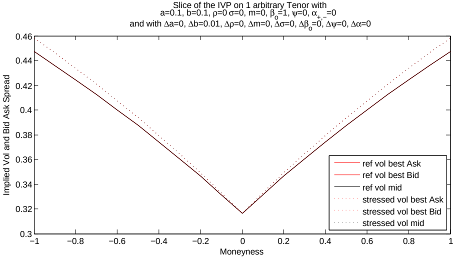
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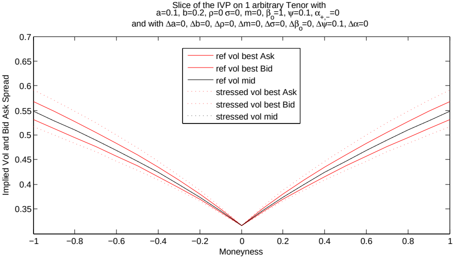
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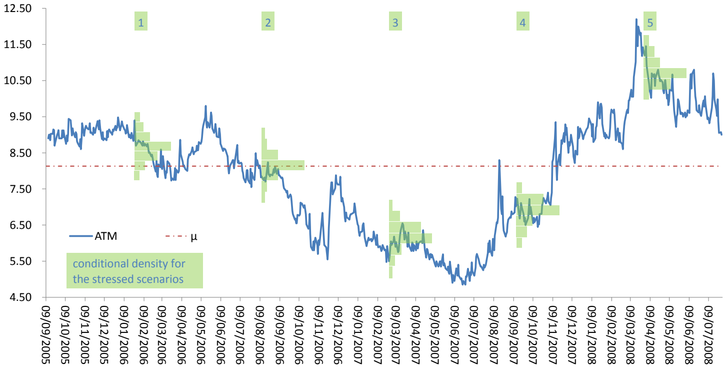
Clustering for Distribution & Regime Change Forecasting

We show how **clustering can help enhance MF** and therefore the two fields can be apposed instead of opposed in the context of modelling risk factors (RF) which behave elements of mean reversion (Spread, Options RF). More specifically we look at how **we can free oneself with the assumptions of SDEs** to construct a general clustering methodology. This can allow us to construct concepts like the **Anticipative VaR** (a leading regime change) as opposed to Responsive VaR (a lagging regime change).

Clustering for Distribution & Regime Change Forecasting



In the left figure, we apply clustering in order to classify **dynamic zones** in which the **returns act differently**. For example when the underlier is significantly above its mean, the forecasted distribution is normally distributed with however a negative mean (vice versa when the underlier is below its historical mean). The distribution is symmetric when at the long term mean.

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Clustering for Distribution & Regime Change Forecasting

In order to **Reconcile** discordant instructions of our regulators to create a risk measure which is **responsive** but **stable** at the same time we propose the **Responsible VaR**, a risk measure responsive on the upside but stable on the downside. We give a couple of examples (figures on the left) of complex portfolio (straddle) backtests in which we modify the λ to control the stability on the downside.

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$$\tilde{\nu}_0^+ = \nu_0^+ \tag{9b}$$

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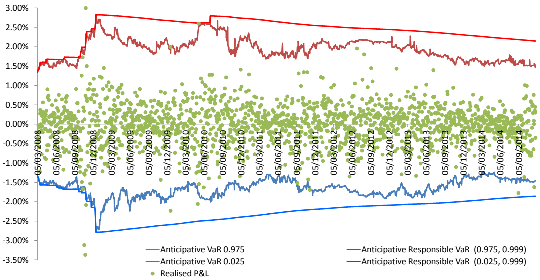
Clustering for Distribution & Regime Change Forecasting



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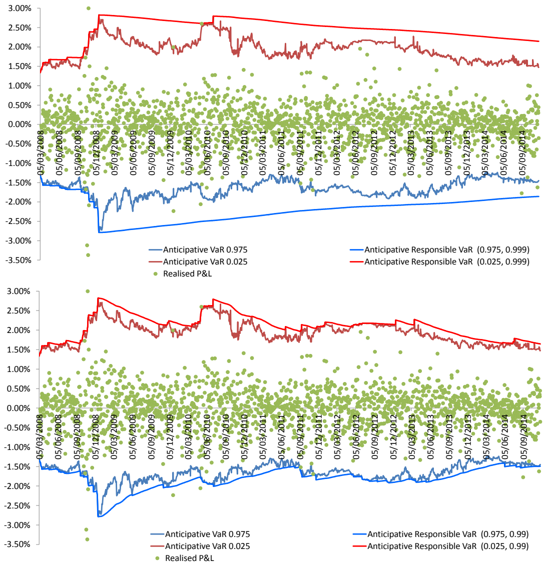

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- Finally we looked at tracking methods using **MTT**.

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- **Is TF the most "moral" strategy**? TF is similar to the TFT (it replicates but adapts).

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- Enhanced the SVI [24, 22] with the **IVP** model [22, 26] designed to adjust exposed data driven limitation of the latter (**wings and liquidity**).

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- **Additional Liquidity issues** for Implied Volatility (the ATM Bid Ask is asset class sensitive) which makes the model less universal.

Future Research (Part II: Apposition)

- **Harmonizing Stochastic & Local Volatility**: We have seen that both the Heston and SVI models are popular in the industry and converge asymptotically to each other [13]: see Equation (10). **Are their limitations linked**?

$$dS_t = \sqrt{v_t} S_t dW_t^1, \qquad S_0 \in \mathbb{R}_+^* \tag{10a}$$

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- **IVP and Assumed Correlation** of Equation (11) the answer?

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To that extend we know that **all models are wrong, but some are useful**[1] and in that spirit we have arguably done the **right thing wronger**[2] in the **1st** part of the thesis but the **wrong thing righter** in the **2nd** part.

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