Data-Driven Models & Mathematical Finance: Opposition or Apposition?
DPhil in Machine Learning Viva Voce

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  - Original Contribution

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- Dynamic of the Financial Market
- Stability of Financial Systems and Multi-Target Tracking

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- Anomaly Detection & Volatility Surface de-Arbitraging
- Big Data Changing the Vanilla Options Landscape
- Clustering for Distribution & Regime Change Forecasting

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- A Bottom-Up Approach to the Financial Markets
- Model Assuming Data vs Data Reassuming the Models
**Research Interest vs Publication**

- Some dedicate their whole lives to **one complicated problem** with a rare (but often nothing) outstanding outcome (Grigori Perelman).
- Some other prefer adhering to the “**publish or perish**” model at the cost of not producing the same quality research.
- **Terence Tao** suggests to have one big problem to go back to when inspired but adhere to (or at least do not neglect) the “publish or perish” model most of the time.
Successful Research Strategies

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Research Complexity vs Recognition

- You can dedicate lots of energy on **problems you find personally stimulating** but nobody cares about.
- You can dedicate little energy on problems you do not find personally very stimulating but **others** may find useful.
- **John Conway**’s Game of Life happens to be the latter case (surprisingly).
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Theory vs Simulation

- A good theory should be able to be simulated.
- A good simulation may change/iron out a theory.
- Cedric Villani thinks that the process can go back and forth until the picture becomes clearer.
Memorable Events Influencing Research

- How does **Morality** emerge in human behaviour → Axelrod [3, 4]:

  - a) Prisoner’s Dilemma
  - b) TFT strategy
  - c) Probabilistic Representation
  - d) War & Peace Chart
  - e) Example of Few Strategy Battles
Memorable Events Influencing Research

How does **Morality** Emerge in human behaviour → Axelrod [3, 4]:

- **a) Prisoner’s Dilemma**
  - C → D
  - \[
    \begin{array}{cc}
      \text{C} & \text{D} \\
      2 & 0 \\
      \end{array}
    \]
    - C
    - D
    - =1
    - =2

- **b) TFT strategy**
  - C → D
  - \[
    \begin{array}{cc}
      \text{C} & \text{D} \\
      0 & 1 \\
      \end{array}
    \]
    - C
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- **c) Probabilistic Representation**
  - \[
    \begin{array}{cc}
      \text{C next} & \text{D next} \\
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      \end{array}
    \]
    - q
    - p

- **d) War & Peace Chart**
  - ALLC → GTFT → ALLD
  - TFT
  - GTFT
  - ALLC
  - ALLD

- **e) Example of Few Strategy Battles**
  - WSLS: CDDDDDDDDDD ...
  - ALLC: CCCCCCCCCCC ...
  - WSLS: CDCDCDCDCD ...
  - WSLS: CDCDCDCDCD ...
  - WSLS: CDCDCDCDCD ...
  - ALLD: DDDDDDDDDDD ...
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  c) Probabilistic Representation
  
  \[ p \text{ TFT} = (1,0) \]

  \[ q \text{ GTFT} = (1,1/3) \]

  d) War & Peace Chart
  
  \[ \text{ALLC} (0,1) \]

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Part II:

- Shocking work experience and the necessity to exude that frustration with papers and vulgarization
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The Rise of Big Data

- **What is Big Data**: lots of anecdotal claims about how big is Big Data [9, 2, 16] but the term refers more to the concept of “**datafication**” (increase in size ≠ better confidence interval but rather change in perspective).
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- The Flash Crashes (eg: [29]) calls for a modelling revolution [5, 12, 6] (BU vs. TD): the Brownian motion assumption to model markets is increasingly difficult to defend.
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- Reconciling discordant Risk: **Anticipative Responsible VaR**.
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Two simple questions for two simple definitions

Definition (Top-Down Vs Bottom-Up)

**Top-Down (TD):** Any *stochastic* quantitative approach which assumes that the *market is random* (or close to random) but for which we can create interesting *dynamic* strategies.
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Can we use this new angle to “solve” the market?
Question: What do we mean by “solve”?
The Scientific Method for “solving” the market

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Answer: We want to look at market price processes and be able to tell what systematic strategies were involved, what their P&L has been and this at all times (including in the future): ecosystem details.
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**Caveat**: Is the idea mad, ambitious or both?
The Scientific Method for “solving” the market

Scientific Method: A good theory can be simulated but simulations can also help bring intuition on what the theory might be [34].

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biological ecosystems **predator/prey** (PP) [35, 8] models \( (a, b, c, d, e, f \text{ and } g \text{ are rate of growth or predation}) \). The relationship between \( x(t), y(t) \) and \( z(t) \) is **deterministic**:

\[
\begin{align*}
\frac{dx(t)}{dt} &= ax(t) - bx(t)y(t) \\
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Predator/Prey models

We can make the **hypotheses that the economical cycles** or oscillation in prices are due to the same type of disruptions that can occur in biological ecosystems **predator/prey (PP)** [35, 8] models ($a, b, c, d, e, f$ and $g$ are rate of growth or predation). The relationship between $x(t)$, $y(t)$ and $z(t)$ is **deterministic**:

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Answering if an ecosystem (or by extension financial market) composed of 3 strategies is stable would come to studying the Jacobian matrix $J$ [8]. If if all eigenvalues of $J(x, y, z)$ have negative real parts then our system is asymptotically stable. Though simplistic, the model can easily be expanded to more complex ecological niches.

\[
J(x, y, z) = \begin{bmatrix}
  a - by & -xb & 0 \\
  yd & -c + dx - ez & -ye \\
  0 & -zg & -f + gy
\end{bmatrix}
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Evolutionary Dynamics

a) Prisoner’s Dilemma

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<th>Previous</th>
<th>C</th>
<th>D</th>
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<tr>
<td>D</td>
<td>D next</td>
<td>D next</td>
</tr>
</tbody>
</table>


c) Probabilistic Representation

- ALLC: (0, 1)
- GTFT: (1, 1/3)
- TFT: (1, 0)


d) War & Peace Chart

- TFT
- GTFT
- ALLD
- ALLC
- rand
- WSL


e) Example of Few Strategy Battles

- WSL: CCCCCCDDDDD ...
  - ALL: CCCCCCCCCC ...
  - GTFT: CCCCCCCCCC ...
  - TFT: CCCCCCCCCC ...

- WSL: CCCCCCDDDCC ...
  - WSL: CCCCCCDCC ...
  - ALLC: CCCCCCCCCC ...
  - ALLD: DDDDDDDDDD ...

- WSL: CDCDCDCDCD ...
  - ALLD: DDDDDDDDDD ...
  - TFT: CCCCCCDCDCDCD ...
  - TFT: CCCCCCDCDCDCD ...

- TFT: CCCCCDCCCCC ...
  - GTFT: CCCCCCCCCC ...
  - GTFT: CCCCCCCCCC ...
  - ALL: CCCCCCCCCC ...

The methodology in ED is interesting because the strategies are both systematic & interacting with each other (like it is the case in algo trading).
A first application in Economics: Minority Game

In the **Minority Game** [7], developed by Challet, Marsili and Zhang, players need to choose between two options (+1, −1). Those who have selected the option chosen by the minority “win”.

---

For large values of $\sigma$, the variance $\sigma$ of the winning group is related to the typical size of the losing group, so the smaller $\sigma$ is, the smaller the variance will be as a function of the parameters of the model $m$. The variance $\sigma$ can be approximated by the sum of the individual variances $\sigma_i$ of the $i$th agent.

The variance $\sigma$ can be calculated using the following formula:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i} \sigma_i^2}$$

where $N$ is the number of agents and $\sigma_i$ is the variance of the $i$th agent.

For $\sigma \geq 1$, the variance $\sigma$ is only a function of the number of agents $N$. However, when $\sigma < 1$, the variance $\sigma$ is also a function of the control parameter $\alpha$.

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$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i} \sigma_i^2}$$

where $N$ is the number of agents and $\sigma_i$ is the variance of the $i$th agent.

When two strategies have the highest number of points, the best strategy is chosen by coin tossing. At low values of $\alpha$, it was found by extensive simulations that the winning group is close to $N/2$, while at high values of $\alpha$, the winning group is close to 1. This finding not only identifies the control parameter in this model, but also paves the way for the application of tools of statistical mechanics in the thermodynamic limit.
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<table>
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<tbody>
<tr>
<td>−1 −1 −1</td>
<td>−1</td>
</tr>
<tr>
<td>−1 −1 +1</td>
<td>−1</td>
</tr>
<tr>
<td>−1 +1 −1</td>
<td>+1</td>
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<tr>
<td>−1 +1 +1</td>
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The fact that this personal ranking can change over time makes the agents adaptive: the ranking of each agent’s strategies can change over time and then serve only to rank strategies within each agent set. After time $t = 1$, $\mu(t) = 4$.

When two strategies have the highest number of points, the best strategy is chosen by coin tossing.
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<td>−1</td>
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<tr>
<td>−1 +1 −1</td>
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<td>−1</td>
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...−1 +1 +1 −1 −1 +1 +1

**Physical Laws** can emerge from these simple rules. We can observe that $\sigma^2/N$ is only a function of $\alpha = 2m/N$ which considering the complexity of the interactions between the set of agents can be quite remarkable.
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<tr>
<td>−1 +1 −1</td>
<td>+1</td>
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<tr>
<td>−1 +1 +1</td>
<td>−1</td>
</tr>
<tr>
<td>+1 −1 −1</td>
<td>−1</td>
</tr>
<tr>
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<td>−1</td>
</tr>
<tr>
<td>+1 +1 +1</td>
<td>+1</td>
</tr>
</tbody>
</table>

**Criticism:** Is this realistic for Economics? Maybe, but not for algorithmic trading (eg: TF strategy in a TF concentrated ecosystem)?

Physical Laws can emerge from these simple rules. We can observe that $\sigma^2/N$ is only a function of $\alpha = 2^m/N$ which considering the complexity of the interactions between the set of agents can be quite remarkable.
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- Foreword
- Historical Context
- Original Contribution

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- Model Assuming Data vs Data Reassuming the Models
As ML “translation” of ED and PP models, we have **Generative Adversarial Networks** (GANs) [14], introduced in 2014, usually involve a system of two neural networks competing in a zero-sum game settings. This process continues as long as needed since the lack of data is no longer a problem.
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The communication tool is the order book and the game is not zero-sum game in our research.
Bringing these Ideas into Electronic Trading

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- The communication tool is the order book and the game is not zero-sum game in our research. The strategies are in **High Frequency Financial Funnel** (HFFF) format [23].
HFFF can model financial strategies:

1. Trend Following (TF)
2. MACD
3. MLR
4. XOR
5. Lasso Regressions etc...

Architecture Complexity and strategy sophistication explains the incentive for Deep Learning (DL). Paradoxically we witness potential for regularization as the network becomes more complex.
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Genetic Algorithm & NN Complexity

The TF (bottom) is less complex than the MLR (middle) which is less complex than the XOR (top) strategy.

Question: Can we make an analogy to the predator-prey ecosystem? Do we get similar behavior as the Lotka-Volterra equations [35, 19]?
Genetic Algorithm & NN Complexity

NN depth & breadth should contribute in increasing the learning potential.

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The Path of Interaction

**Answer**: not quite but there are **few interesting links** (exponential grow of the smaller prey/self fulfilling strategies such as TF) but they are **many issues** (classification, timescale etc...).
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The genetic algorithm presented in the previous slide creates complications (classification issues). This pushed us to study the bottom-up approach using concepts taken from evolutionary dynamics and created the concept of **Path of Interaction**: table of 7 rows documenting the interaction’s details (eg: table above).
The Path of Interaction

Answer: not quite but there are few interesting links (exponential grow of the smaller prey/self fulfilling strategies such as TF) but they are many issues (classification, timescale etc...).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>seed ↑</th>
<th>TF1</th>
<th>TF2</th>
<th>TF1</th>
<th>TF2</th>
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<tr>
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<td></td>
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</tr>
<tr>
<td>Signal</td>
<td>N/A</td>
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<td>+1</td>
<td>+1</td>
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<tr>
<td>OB</td>
<td>0P₁,₁,₁,₁</td>
<td>0,0P₁,₁,₁</td>
<td>0,0,0P₁,₁</td>
<td>0,0,0,0P₁</td>
<td></td>
</tr>
<tr>
<td>Last Price</td>
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<td>101</td>
<td>102</td>
<td>103</td>
<td>104</td>
</tr>
<tr>
<td>ΔOI</td>
<td>+1</td>
<td>−1</td>
<td>−2</td>
<td>−3</td>
<td></td>
</tr>
<tr>
<td>ΔPrice</td>
<td>+1</td>
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<td>+1</td>
<td>+1</td>
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<td>[0,0]</td>
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<td>[2,1]</td>
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Sequential Monte Carlo (SMC) methods [10, 11, 18], also know as Particle Filter have emerged as a fashionable tool to track scenarios in the last 15 years [31, 32]. They are the sequential analogue of Markov Chain Monte Carlo (MCMC) methods and similar to importance sampling methods.

The aims of the PF is to estimate the sequence of hidden parameters (eg: the frequencies of certain types of strategies), based on an indirect observations (eg: the fluctuations of the market).

Resampling methods are usually used to avoid the problem of weight degeneracy in our algorithm. Avoiding situations where our trained probability measure tends towards the Dirac distribution must be avoided because it really does not give much information on all the possibilities of our state. There exists many different resampling methods, Rejection Sampling, Sampling-Importance Resampling, Multinomial Resampling, Residual Resampling, Stratified Sampling, and the performance of our algorithm can be affected by the choice of our resampling method. The stratified resampling proposed by Kitagawa [9] is optimal in terms of variance. Figure 2.3 gives an illustration of the Stratified Sampling and the corresponding algorithm is described in algorithm 13.

**CDF**

$$F(U) = \begin{cases} 
N_p^{-1}, & 0 < U < \frac{1}{N_p} \\
1 - \frac{1}{N_p}, & \frac{1}{N_p} < U < 1 \end{cases}$$

**X**

$$X_k$$

**U**

$$U_1 \sim \mathcal{N}(0, \frac{1}{N_p})$$

$$U_2 \sim \mathcal{N}(\frac{1}{N_p}, \frac{2}{N_p})$$

$$U_{N_p} \sim \mathcal{N}(\frac{N_p - 1}{N_p}, 1)$$

**Sampling**

- **Estimated pdf at time k (before resample)**
- **Real pdf**

**Resampling**

- Another seemingly useless particle is relocated as expected at a more useful place.
- Another useless particle stays at the same spot.

**Estimated pdf at time k+1 (after resample)**
Sequential Monte Carlo (SMC) methods [10, 11, 18], also known as Particle Filter, have emerged as a fashionable tool to track scenarios in the last 15 years [31, 32]. They are the sequential analogue of Markov Chain Monte Carlo (MCMC) methods and similar to importance sampling methods.

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The figure illustrates the Stratified Sampling. We see a particle sampled from the real distribution at time $k$, which is then resampled to a new location at time $k+1$. The lucky useless particle stays at the same spot, while another seemingly useless particle is relocated as expected at a more useful place.
The random Brownian paths become deterministic.

Each deterministic path corresponds to a sequence of interaction of several strategies for which the sequence and the P&Ls can be traced through our SMC methods by looking at the market price only.

2 Strategies Ecosystems: $[s_1, s_2, \ldots, s_9]$

3 Strategies Ecosystems: $[s_{10}, s_{11}, \ldots, s_{15}]$
The random Brownian paths become deterministic

Each **Brownian Path Lookalike** is no longer stochastic but **deterministic**.
The random Brownian paths become deterministic

Each **Brownian Path Lookalike** is no longer stochastic but **deterministic**.

Each **deterministic path** corresponds to a **sequence of interaction** of several **strategies** for which the sequence and the **P&Ls** can be traced through our SMC methods by looking at the market **price only**.
PF assigns a probability for each ecosystem scenario

We have recorded 15 different scenarios (ecosystem history) for the sake of this presentation, all of which are clearly detected after the 11th iteration [27].
PF assigns a probability for each ecosystem scenario

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![Bar charts representing different scenarios](image-url)
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Model Assuming Data

**Definition (Correlation Model)**

On a filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F})_{t \geq 0}, \mathbb{P})\) the Correlation model is given by the set of two SDE’s:

\[
\begin{align*}
    dX(t) &= \mu X(t) dt + \sigma X(t) dW(t), \\
    dY(t) &= \mu Y(t) dt + \sigma Y(t) d\tilde{W}(t), \\
    d\langle W, \tilde{W} \rangle_t &= \rho dt,
\end{align*}
\]

\(1\)

where \(\mu \in \mathbb{R}, \sigma > 0\) are drift and diffusion coefficients of asset price, \(\tilde{W}(t)\) and \(W(t)\) are two correlated Brownian motions with constant correlation coefficient \(\rho \in [-1, 1]\).
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Remark: few assumptions need to be fulfilled. Variance must remain constant (homoscedasticity) and the returns independent. If not instantaneous measured correlation is misleading with respect to long term correlation:
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- Top Right Figure: \(\hat{\rho} = -1\),
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\text{(1)} \quad &dX(t) = \mu X(t) dt + \sigma X(t) dW(t), \\
\text{(2)} \quad &dY(t) = \mu Y(t) dt + \sigma Y(t) d\tilde{W}(t),
\end{align*}
\]

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- Middle Right Figure \(\hat{\rho} = 0\),
Model Assuming Data

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- Top Right Figure: \(\hat{\rho} = -1\),
- Middle Right Figure \(\hat{\rho} = 0\),
- Bottom Right Figure \(\hat{\rho} = +1\).
Data Disagrees with Assumptions

Data violates assumptions of the classic model (2):

- **directly** (this is not new: the returns are not iid and the variance is not constant),
Data violates assumptions of the classic model (2):

- **directly** (this is not new: the returns are not iid and the variance is not constant),
- **indirectly** (Figure on left hand side). Implied correlation is “marked”, the same way implied volatility is. The ratio of two closely related underliers exhibits mean reversion in the minds of the traders risk managing these products.
Data Reassuming the Model

**Intuitive Definition:** Cointelation is a portmanteau neologism in finance, designed to signify a hybrid method between between *cointegration* and correlation models (Data-Driven adjustment to classic financial math models: data “reassuming the model”).

**Definition (Cointelation Model):** On a filtered probability space \((\Omega, F, (F_t)_{t \geq 0}, P)\), the Cointelation model is given by the SDE’s:

\[
\begin{align*}
    dX(t) &= \mu X(t) \, dt + \sigma X(t) \, dW(t), \\
    dY(t) &= \kappa (X(t) - Y(t)) \, dt + \eta Y(t) \, d\tilde{W}(t), \\
    d\tilde{W}(t) &= \rho dt,
\end{align*}
\]

with \(\mu \in \mathbb{R}, \sigma > 0, \eta > 0\) are drift and diffusion coefficients of asset price \(X\), \(\kappa\) the rate of mean reversion, \(\tilde{W}(t)\) and \(W(t)\) are two correlated Brownian motions with constant correlation coefficient \(\rho \in [-1, 1]\). The process \((X(t))_{t \geq 0}\) is called the leading process, \((Y(t))_{t \geq 0}\) the lagging process.

Remark: We note that setting \(\kappa = 0\) yields the classic correlation model. Conversely, setting \(\rho = 0\) yields the cointegration model.
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On a filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})\) the Cointelation model is given by the SDE’s:

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  d\langle W, \tilde{W} \rangle_t &= \rho dt,
\end{align*}
\]

with \(\mu \in \mathbb{R}, \sigma > 0, \eta > 0\) are drift and diffusion coefficients of asset price \(X\), \(\kappa\) - the rate of mean reversion, \(\tilde{W}(t)\) and \(W(t)\) are two correlated Brownian motions with constant correlation coefficient \(\rho \in [-1, 1]\). The process \((X(t))_{t \geq 0}\) is called the leading process, \((Y(t))_{t \geq 0}\) the lagging process.

Intuitive Definition: Cointelation is a portmanteau neologism in finance, designed to signify a hybrid method between between cointegration and correlation models (Data-Driven adjustment to classic financial math models: data “reassuming the model”).

Cointelation Model

Correlation Model
Cointegration Model
**Definition (Cointelation Model)**

On a filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F})(t\geq 0), \mathbb{P})\) the Cointelation model is given by the SDE's:

\[
\begin{align*}
    dX(t) &= \mu X(t) dt + \sigma X(t) dW(t), \\
    dY(t) &= \kappa (X(t) - Y(t)) dt + \eta Y(t) d\tilde{W}(t), \\
    d(W, \tilde{W})_t &= \rho dt,
\end{align*}
\]

with \(\mu \in \mathbb{R}, \sigma > 0, \eta > 0\) are drift and diffusion coefficients of asset price \(X\), \(\kappa\) - the rate of mean reversion, \(\tilde{W}(t)\) and \(W(t)\) are two correlated Brownian motions with constant correlation coefficient \(\rho \in [-1, 1]\). The process \((X(t))_{t\geq 0}\) is called the leading process, \((Y(t))_{t\geq 0}\) the lagging process.

**Remark**: We note that setting \(\kappa = 0\) yields the classic correlation model. Conversely, setting \(\rho = 0\) yields the cointegration model.

**Intuitive Definition**: Cointelation is a portmanteau neologism in finance, designed to signify a hybrid method between between cointegration and correlation models (Data-Driven adjustment to classic financial math models: data “reassuming the model”).
Interesting Property: the Inferred Correlation

Definition (Inferred Correlation)

Considering the dynamics of equation (3), the Inferred Correlation formula is given by (3).

\[ \rho^* \tau \approx \rho + (1 - \rho) \left[ 1 - \exp(-\kappa \lambda (\tau - 1)) \right] \]  

where \( \rho^* \tau \) = \( E \left[ \sup_{0 < t \leq \tau} \rho_t \right] \), \( \tau \in \mathbb{Z}^* \), \( \theta \in [0, 1] \) and \( \lambda \) constant.

Remark: Cointelation model can hit the whole correlation spectrum depending on timescale.
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**Remark:** Cointelation model can hit the whole correlation spectrum depending on timescale.
Socially Responsible & Consumer Finance is a wave of quant finance that gains momentum after each crisis but quickly runs unfortunately out of fuel:

- **Situation**: Equity/Commodities salesman trying to convince clients who have a portfolio in the Commodities/Equities asset class to diversify by buying salesman’s products.

- **Objective**: show small correlation to suggest diversification benefits
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---

![Graph of Normalised plot of Oil and BP since April 2007](image)

![Graph of Measured correlation of the above time series at different time horizon](image)
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- **Solution**: Inferred Correlation.
We consider the portfolio of:

- **two stocks** $X_t$ and $Y_t$
  with price dynamics following the cointelation model (3).

The optimization process involves a dynamics switching strategy between both approaches:

$$w^*_t = \max_h \left( \frac{\frac{1}{\tau} \mathbb{E}(r_p) - \sigma^2(r_p)}{\frac{1}{\tau} \mathbb{E}(V_\pi(T)) \gamma} \div \frac{\frac{1}{\tau} \mathbb{E}(X_t - Y_t)}{\frac{1}{\tau} \mathbb{E}(X_t - Y_t)} \right) \div \mu + \sup_{\pi(t) \in A(0, \nu_0)} \frac{\gamma \mathbb{E}(V_\pi(T)) \gamma}{\frac{1}{\tau} \mathbb{E}(X_t - Y_t)} \div \mu$$

The red part does not have a closed form solution: the pure Classic Financial Mathematics does not have a solution.
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We can notice that we can decompose the cointelation model in two familiar models:

- Markowitz Mean-Variance Analysis ($\rho$-centered sub strategy): Easy
- Ornstein−Uhlenbeck "like" Stochastic Control ($\kappa$-centered sub strategy): Difficult

The optimization process involves a dynamics switching strategy between both approaches

\[
W^*_t = \max h(t)/2 \tau E(r_p) - \sigma^2(r_p)/2 \kappa (X_t - Y_t)/Y_t > \mu + \sup_{\pi(t) \in A(0, v_0)} E[1 \gamma(V_\pi(T))]/\kappa (X_t - Y_t)/Y_t \leq \mu
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\[ w^* \geq \max_h \frac{1}{\tau E(r_p - \sigma^2(r_p)/\kappa(X_t - Y_t/Y_t))} \geq \mu + \sup_{\pi \in A(0, v_0)} E[\gamma(V_\pi(T))]/\kappa(X_t - Y_t/Y_t)] \]

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$$
W_t^* = \max_{h(t)} \left[ 2\tau E(r_p) - \sigma^2(r_p) \right] 1\left\{ \kappa \left( \frac{X_t - Y_t}{Y_t} \right) \leq \mu \right\} + \\
\sup_{\pi(t) \in \mathcal{A}(0,v_0)} \mathbb{E} \left[ \frac{1}{\gamma} (V^\pi(T))^\gamma \right] 1\left\{ \kappa \left( \frac{X_t - Y_t}{Y_t} \right) > \mu \right\}
$$

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$$w_t^* = \max_{h(t)} \left\{ \frac{2\tau E(r_p) - \sigma^2(r_p)}{\exp\left\{ \frac{1}{\gamma} (V^\pi(T))^{\gamma} \right\}} \right\}$$

The **red part** does not have a closed for solution: the pure Classic Financial Mathematics does not have a solution.
The way to handle the red part part is to consider the function $G(t, v, s)$ such that $G \in C^{1,2}(Q)$, the Hamilton-Jacobi-Bellman (HJB) equation corresponding to stochastic control problem is

$$\frac{\partial G}{\partial t}(t, v, s) + \sup_{\pi} \mathcal{L}^{\pi} G(t, v, s) = 0,$$  \hspace{1cm} (4)
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![Diagram](image.png)
We can use clustering in order to study the P&L of \( n \) (eg: 4) strategies through time.
Portfolio Optimization: Pure ML Approximation

We can use clustering in order to study the P&L of $n$ (eg: 4) strategies through time.
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- **Strategy $S^{++}$** in which we are long both $X$ and $Y$ at time $t$ within bands $[a_i, b_i]$, $i \in \mathbb{N}$, and with P&L $V^{++}_{[a_i, b_i], t}$.
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- **Strategy $S^{+-}$** in which we are long $X$ and short $Y$ at time $t$ within $[a_i, b_i]$, $i \in \mathbb{N}$, and with P&L $V^{+-}_{[a_i, b_i], t}$.
Portfolio Optimization: Pure ML Approximation

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We can use clustering in order to study the P&L of $n$ (eg: 4) strategies through time.

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- **Strategy $S^{--}$** in which we are short both $X$ and $Y$ at time $t$ within bands $[a_i, b_i]$, $i \in \mathbb{N}$, and with P&L $V^{--}_{[a_i, b_i], t}$.

\[\begin{align*}
\text{Figure 5: Comparative study of cointelated and correlated pairs through simulation of a very strong.}
\end{align*}\]
We can use clustering in order to study the P&L of \( n \) (eg: 4) strategies through time.

- **Strategy \( S^{++} \)** in which we are long both \( X \) and \( Y \) at time \( t \) within bands \( [a_i, b_i], \quad i \in \mathbb{N} \), and with P&L \( V^{++}[a_i, b_i], t \).
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**Remark:** We can have as many bands (strategies) as we have weight proportions.
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Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left figure)
Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left figure) in rolling contract form

Increased uncertainty about where the data point is for which there is currently no proposed methodology.
Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left figure) in rolling contract form into a coherent fixed pillars implied volatility surface (bottom right figure) presents challenges.

Increased uncertainty about where the data point is for which there is currently no proposed methodology.
The classic arbitrage conditions (butterfly & calendar spread) have been replaced with more elegant models (e.g. \( \forall K, \forall T, |T \partial_K \sigma^2(K, T)| \leq 4 \)) that were nevertheless not sufficient and the arrival of Big Data in the wings exposed these limitations in the Financial Mathematics models.
The classic arbitrage conditions (butterfly & calendar spread) have been replaced with more elegant models (e.g. $\forall K, \forall T, |T \partial K \sigma^2(K, T)| \leq 4$) that were nevertheless not sufficient and the arrival of Big Data in the wings exposed these limitations in the Financial Mathematics models.

Remark 3.2. By a careful study of the minima and the shapes of the two slices $w(\cdot, t_1)$ and $w(\cdot, t_2)$, it is possible to determine a set of conditions on the parameters ensuring no calendar spread arbitrage. However these conditions involve tedious combinations of the parameters and will hence not match the computational simplicity of the lemma.

For a given slice, we now wish to determine conditions on the parameters of the raw SVI formulation (3.1) such that butterfly arbitrage is excluded. By Lemma 2.1, this is equivalent to showing (i) that the function $g$ defined in (2.1) is always positive and (ii) that call prices converge to zero as the strike tends to infinity. Sadly, the highly non-linear behaviour of $g$ makes it seemingly impossible to find general conditions on the parameters that would eliminate butterfly arbitrage. We provide below an example where butterfly arbitrage is violated. Notwithstanding our inability to find general conditions on the parameters that would preclude arbitrage, in Section 4, we will introduce a new sub-class of SVI volatility surface for which the absence of butterfly arbitrage is guaranteed for all expiries.

Example 3.1. (From Axel Vogt on wilmott.com) Consider the raw SVI parameters: $(a, b, m, \rho, \sigma) = (-0.0410, 0.1331, 0.3586, 0.3060, 0.4153)$, (3.8)

with $t = 1$. These parameters give rise to the total variance smile $w$ and the function $g$ defined in (2.1) on Figure 1, where the negative density is clearly visible.

Figure: Vogt's total variance example verifying $b(1 + |\rho|) \leq \frac{4}{T}$ (left figure: with the x axis being the log-moneyness and the y axis being the implied variance) and the corresponding $\partial^2_{K,K} BS(\sigma^2(K, T))$ approximating the (supposed) always positive pdf (right figure: with the x axis being the log-moneyness and the y axis being the non normalized pdf).
The work of this chapter revolves around anomaly detection in the context of the implied volatility surface trying to use the classic methods: butterfly arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the more advanced methods such as in equation (5e) (a necessary but not sufficient condition).
The work of this chapter revolves around anomaly detection in the context of the implied volatility surface trying to use the classic methods: butterfly arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the more advanced methods such as in equation (5e) (a necessary but not sufficient condition).

solve: \( \hat{\sigma}_t(\tau, d) = \arg \min_{\tilde{\sigma}_t(\tau, d)} \sum_{\tau} \sum_{d} \left[ C(\sigma_{i, t}(\tau, d)) - C(\tilde{\sigma}_t(\tau, d)) \right]^2 \) (5a)

subject to: \( \forall \tau \) and \( \forall K \)

\[
C(K - \Delta, \sigma_0(K - \Delta, \tau)) - C(K, \sigma_0(K, \tau)) \geq 0 \quad (5b)
\]

\[
C(K, \tau + \Delta, \sigma_0(K, \tau + \Delta)) - C(Ke^{-r\Delta}, \sigma_0(Ke^{-r\Delta}, \tau)) \geq 0 \quad (5c)
\]

but not:

\[
\forall K, \forall T, |T \partial_K \sigma^2(K, T)| \leq 4 \quad (5d)
\]
The work of this chapter revolves around **anomaly detection** in the context of the implied volatility surface trying to use the **classic methods**: butterfly arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the **more advanced methods** such as in equation (5e) (a **necessary but not sufficient** condition). We also discuss some of the **idiosyncratic asset class related differences** that require modifications in the optimization process. Finally we discuss few **trading idea** by simplifying the IVP model.

\[
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\]

subject to: \(\forall \tau \) and \(\forall K\)

\[
C(K - \Delta, \sigma_0(K - \Delta, \tau)) - C(K, \sigma_0(K, \tau)) \geq 0
\]

\[\tag{5b}\]

\[
C(K, \tau + \Delta, \sigma_0(K, \tau + \Delta)) - C(K e^{-r\Delta}, \sigma_0(K e^{-r\Delta}, \tau)) \geq 0
\]

\[\tag{5c}\]

berry arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the **more advanced methods** such as in equation (5e) (a **necessary but not sufficient** condition). We also discuss some of the **idiosyncratic asset class related differences** that require modifications in the optimization process. Finally we discuss few **trading idea** by simplifying the IVP model.

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\]

subject to: \(\forall \tau \) and \(\forall K\)

\[
C(K - \Delta, \sigma_0(K - \Delta, \tau)) - C(K, \sigma_0(K, \tau)) \geq 0
\]

\[\tag{5b}\]

\[
C(K, \tau + \Delta, \sigma_0(K, \tau + \Delta)) - C(K e^{-r\Delta}, \sigma_0(K e^{-r\Delta}, \tau)) \geq 0
\]

\[\tag{5c}\]

but not:

\[
\forall K, \forall T, |T \partial_K \sigma^2(K, T)| \leq 4
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\[\tag{5e}\]
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Big Data Changing the Vanilla Options Landscape

- exposed the limitations of both the **wings** (e.g. **SVI**) and of the **liquidity** in the options market,

We saw in the **Options market**:
- a step back in **complexity** (from exotics to vanilla) but with more **coherent** pricing
- a step forward in **liquidity** modelling
Big Data Changing the Vanilla Options Landscape

- exposed the limitations of both the wings (e.g. SVI) and of the liquidity in the options market,
- the need for proxying when the data is scarce.

We saw in the Options market:
- a step back in complexity (from exotics to vanilla) but with more coherent pricing
- a step forward in liquidity modelling
- the need (in CCPs), to map risk factors to economical concepts.
The rise of big data:
- exposed the limitations of both the wings (e.g. SVI) and of the liquidity in the options market,
- the need for proxying when the data is scarce.
- the Heston (Stochastic Volatility) model and the local volatility model and the need for harmonizing these two concepts.

We saw in the Options market:
- a step back in complexity (from exotics to vanilla) but with more coherent pricing
- a step forward in liquidity modelling
- the need (in CCPs), to map risk factors to economical concepts.

\[
\begin{align*}
    dS_t &= \sqrt{v_t} S_t \, dW^1_t, \quad S_0 \in \mathbb{R}_+^* \\
    dv_t &= \kappa (\theta - v_t) \, dt + \sigma \sqrt{v_t} \, dW^2_t, \quad v_0 \in \mathbb{R}_+^* 
\end{align*}
\]
Big Data exposed the limitations of the SVI in the wings and the subprime crisis of 2007 exposed the need to incorporate liquidity directly in the options model.
We define the Implied Volatility surface Parametrization (IVP) split with its mid in equation (7) with the downside transform in equation (7b) enhancing the SVI,

$$\sigma^2_{IVP, o, \tau}(k) = a_\tau + b_\tau \left[ \rho_\tau (z_{o, \tau} - m_\tau) + \sqrt{(z_{o, \tau} - m_\tau)^2 + \sigma^2_\tau} \right]$$

(7a)

$$z_{o, \tau} = \frac{k}{\beta_{o, \tau}^{-1+4|k-m|}}$$

(7b)

Big Data exposed the limitations of the SVI in the wings and the subprime crisis of 2007 exposed the need to incorporate liquidity directly in the options model. The IVP addresses these two points.
We define the Implied Volatility surface Parametrization (IVP) split with its mid in equation (7) with the downside transform in equation (7b) enhancing the SVI,

\[ \sigma_{IVP,0,\tau}^2(k) = a_\tau + b_\tau \left[ \rho_\tau (z_{0,\tau} - m_\tau) + \sqrt{(z_{0,\tau} - m_\tau)^2 + \sigma_\tau^2} \right] \]  

and its liquidity parameters in equation (8). Parameters \( \psi \) represents the Wings Curvature, \( \alpha \) represents the ATM Spread. The latter two parameters can be defined in terms of functions to accommodate the position size in which case the market dept is controlled with the \( \eta \) parameters.

\[ \sigma_{IVP,+,-,\tau}^2(k) = a_\tau + b_\tau \left[ \rho_\tau (z_{+,\tau} - m_\tau) + \sqrt{(z_{+,\tau} - m_\tau)^2 + \sigma_\tau^2} \right] + \alpha_\tau(p) \]  

\[ z_{+,\tau} = z_{0,\tau}[1 + \psi_\tau(p)] \]  

\[ \sigma_{IVP,-,+,\tau}^2(k) = a_\tau + b_\tau \left[ \rho_\tau (z_{-,\tau} - m_\tau) + \sqrt{(z_{-,\tau} - m_\tau)^2 + \sigma_\tau^2} \right] - \alpha_\tau(p) \]  

\[ z_{-,\tau} = z_{0,\tau}[1 - \psi_\tau(p)] \]  

\[ \alpha_\tau(p) = \alpha_{0,\tau} + (a_\tau - \alpha_{0,\tau})(1 - e^{-\eta\alpha_\tau p}) \]  

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Big Data exposed the limitations of the SVI in the wings and the subprime crisis of 2007 exposed the need to incorporate liquidity directly in the options model. The IVP addresses these two points as well as allow for additional benefits:

- Proxying
- Backtesting weaponry on complex strategies.
Slice of the IVP on 1 arbitrary Tenor with $a=0.1, b=0.1, \rho=0, \sigma=0, m=0, \beta_0=1, \psi=0, \alpha^+, -, 0$ and with $\Delta a=0.01, \Delta b=0, \Delta \rho=0, \Delta m=0, \Delta \sigma=0, \Delta \beta_0=0, \Delta \psi=0, \Delta \alpha=0$
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Moneyness

Implied Vol and Bid Ask Spread

ref vol best Ask
ref vol best Bid
ref vol mid
stressed vol best Ask
stressed vol best Bid
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We show how clustering can help enhance MF and therefore the two fields can be apposed instead of opposed in the context of modelling risk factors (RF) which behave elements of mean reversion (Spread, Options RF). More specifically we look at how we can free oneself with the assumptions of SDEs to construct a general clustering methodology. This can allow us to construct concepts like the Anticipative VaR (a leading regime change) as opposed to Responsive VaR (a lagging regime change).
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In the left figure, we apply clustering in order to classify dynamic zones in which the returns act differently. For example when the underlier is significantly above its mean, the forecasted distribution is normally distributed with however a negative mean (vice versa when the underlier is below its historical mean). The distribution is symmetric when at the long term mean.
In order to **Reconcile** discordant instructions of our regulators to create a risk measure which is **responsive** but **stable** at the same time we propose the **Responsible VaR**, a risk measure responsive on the upside but stable on the downside. We give a couple of examples (figures on the left) of complex portfolio (straddle) backtests in which we modify the $\lambda$ to control the stability on the downside.

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\[
\alpha = \int_{-\infty}^{\nu_t^+} p_t(x) \, dx \quad \text{(9a)}
\]
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\tilde{\nu}_0^+ = \nu_0^+ \quad \text{(9b)}
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\]
\[
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\]
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- This was done in order to express the bottom-up approach at the infinitesimal level. More specifically we developed the concept of **Path of Interaction** in an **HFTE Game**.
- Finally we looked at tracking methods using **MTT**.
Future Research (Part I: Opposition)

- **Classification Simplification**: the direct simulation approach of an HFTE [23] creates situations in which two very different architectures yield the exact same function.

- **Complex Food Webs**: We need to bring the complexity of our state space to the level of a complex food web. Additional strategies must be incorporated and more HFTE games must be included in our database of scenarios and we need to incorporate Birth & Death Processes.

- **Order-Book Dynamics**: Many of the markets are driven by different rules for the OB.

- **Increased HFFF complexity does not equate to Invasion**: A clear picture did not necessarily emerge from the first simulations.

- **Diversity as it relates to Stability**: In biology diversity in an ecosystem leads to its instability [35, 8] but what about Finance? Is TF the most "moral" strategy? TF is similar to the TFT (it replicates but adapts).
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- Pointed to **errors** in the FM literature when it comes to Implied Volatility **arbitrage** modelling and introduced **de-arbing** method.

- Enhanced the SVI [24, 22] with the **IVP** model [22, 26] designed to adjust exposed data driven limitation of the latter (**wings and liquidity**).
There are few extensions or improvements that can be performed on the optimization process for cointelated pairs research. We can first ask ourselves the question of the **n-Cointelated** case.

- Testing of the IVP proxying methodology with quality data (e.g., does the $\rho_S$ parameter compare well with the $\rho_{SS}$?).
- Application of the above two (Cryptocurrency Option’s market proxy; Bitcoin vs Altcoins) to the world of Cryptocurrency.
- Current De-Arbitraging methodology is not robust in between pillars (e.g., Interpolation).
- Particle Filter for Implied Volatility MTT: very complex co-movements of 3 or more parameters are not taken into account.
- Though, these do not matter for vanilla options, they may matter for more complex exotics.
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Harmonizing Stochastic & Local Volatility: We have seen that both the Heston and SVI models are popular in the industry and converge asymptotically to each other [13]: see Equation (10). Are their limitations linked?

\[
\begin{align*}
    dS_t &= \sqrt{v_t} S_t dW_1^t, \quad S_0 \in \mathbb{R}_+^* \quad (10a) \\
    dv_t &= \kappa(\theta - v_t) dt + \sigma \sqrt{v_t} dW_2^t, \quad v_0 \in \mathbb{R}_+^* \quad (10b) \\
    d \langle W^1, W^2 \rangle_t &= \rho dt, \quad (10c) \\
    v(k, t) &\rightarrow a + b[\rho(k - m) + \sqrt{(k - m)^2 + \sigma^2}] \quad (10d)
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IVP and Assumed Correlation of Equation (11) the answer?

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    dv_t &= \kappa(\theta - \nu_t) dt + \sigma \sqrt{\nu_t} dW_2^t, \quad v_0 \in \mathbb{R}^*_+ \quad (11b) \\
    d\langle W^1, W^2 \rangle_t &= \rho(t, S_t) dt, \quad (11c) \\
    \rho(t, S_t) &= \rho_+(t) + [\rho_-(t) - \rho_+(t)] [1 - \exp(-\beta(t)|S_t - K|)] \quad (11d) \\
    \nu(k, t) &\to a + b[\rho(z - m) + \sqrt{(z - m)^2 + \sigma^2}] \quad (11e)
\end{align*}
The more efficient you are at doing the wrong thing, the wronger you become. If you do the right thing wrong and correct it, you get better.

– Russell L. Ackoff

To that extend we know that all models are wrong, but some are useful\footnote{a quote that is generally attributed to George Box} and in that spirit we have arguably done the right thing wronger\footnote{to mean, less useful for practitioners} in the 1st part of the thesis but the wrong thing righter in the 2nd part.
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To that extend we know that all models are wrong, but some are useful\(^1\) and in that spirit we have arguably done the right thing wronger\(^2\) in the 1st part of the thesis but the wrong thing righter in the 2nd part.

\(^1\) a quote that is generally attributed to George Box
\(^2\) to mean, less useful for practitioners.
Al-Aradi, A., Correia, A., Naiff, D., and Jardim, G.
Solving nonlinear and high-dimensional partial differential equations via deep learning.

Al-Jarrah, O., Yoo, P., Muhaidat, S., Karagiannidis, G., and Taha, K.
Efficient machine learning for big data: A review.
87–93.

Axelrod, R.
The evolution of cooperation.

Axelrod, R.
The complexity of cooperation: Agent-based models of competition and collaboration.

Bouchaud, J.-P.
Economics needs a scientific revolution.
*Nature 455* (2008), 1181.

Buchanan, M.
Meltdown modelling: Could agent-based computer models prevent another financial crisis?

Challet, D., Marsili, M., and Zhang, Y.-C.
*Minority Games: Interacting Agents In Financial Markets.*

Chauvet, E., Paullet, J. E., Previte, J. P., and Walls, Z.
A Lotka-Volterra three-species food chain.

Cukier, K., and Mayer-Schoenberger, V.
The rise of big data: How it’s changing the way we think about the world.

Doucet, A., de Freitas, N., and Gordon, N.
Sequential monte carlo in practice.

**Doucet, A., Godsill, S., and Andrieu, C.**
On sequential monte carlo sampling methods for bayesian filtering.

**Farmer, J. D., and Foley, D.**
The economy needs agent-based modelling.

**Gatheral, J., and Jacquier, A.**
Convergence of Heston to SVI.

Generative adversarial networks.

**Johnson, T. C.**
Finance and mathematics: Where is the ethical malaise?
*The Mathematical Intelligencer 37*, 4 (Dec 2015), 8–11.

**Lee, J.-G., and Minseo, K.**
Geospatial big data: Challenges and opportunities.

**Leonard, C.**
Apres la crise, l’enseignement de la finance repense.

**Liu, J., and Chen, R.**
Sequential monte carlo methods for dynamic systems.
*Journal of the American Statistical Association*.

**Lotka, A. J.**
Elements of physical biology.
*Williams & Wilkins Co.* (1925).
Mahdavi-Damghani, B.
UTOPE-ia.

Mahdavi-Damghani, B.
The non-misleading value of inferred correlation: An introduction to the cointelation model.

Mahdavi-Damghani, B.
Introducing the implied volatility surface parametrisation (IVP): application to the fx market.

Mahdavi-Damghani, B.
Introducing the HFTE model: a multi-species predator prey ecosystem for high frequency quantitative financial strategies.

Mahdavi-Damghani, B., and Kos, A.
De-arbitraging with a weak smile.

Mahdavi-Damghani, B., and Roberts, S.
A proposed risk modeling shift from the approach of stochastic differential equation towards machine learning clustering: Illustration with the concepts of anticipative & responsible VaR.

Mahdavi-Damghani, B., and Roberts, S.
Machine learning techniques for deciphering implied volatility surface data in a hostile environment: Scenario based particle filter, risk factor decomposition & arbitrage constraint sampling.

Mahdavi-Damghani, B., and Roberts, S.
A bottom-up approach to the financial markets agent-based quantitative algorithmic strategies: Ecosystem, dynamics & detection.

Mahdavi-Damghani, B., Welch, D., O’Malley, C., and Knights, S.
The misleading value of measured correlation.  
*Wilmott* 62 (2012), 64—73.

**Nanex.**  
Strange days June 8th, 2011 - natgas algo, 2011.

**Nowak, M.**  
*Evolutionary dynamics: exploring the equations of life.*  
2006.

**Sidenbladh, H.**  
Multi-target particle filtering for the probability hypothesis density.  

**Sidenbladh, H., and Wirkander, S.**  
Particle filtering for finite random sets.  

**Sirignano, J., and Spiliopoulos, K.**  
Dgm: A deep learning algorithm for solving partial differential equations.  

**Villani, C.**  
Cedric villani: la simulation numérique est un enjeu majeur pour la société, 2012.

**Volterra, V.**  
Variazioni e fluttuazioni del numero d’individui in specie animali conviventi.  