Data-Driven Models & Mathematical Finance: Opposition or Apposition?

DPhil in Machine Learning Viva Voce

Babak Mahdavi-Damghani[1]

University of Oxford

[1] *babak.mahdavidamghani@oxford-man.ox.ac.uk*

September 19th, 2019

Successful Research Strategies

Research Interest vs Publication

- Some dedicate their whole lives toone complicated problem with a rare (but often nothing) outstanding outcome (Grigori Perelman).
- Some other prefer adhering to the \publish or perish" model at the cost of not producing the same quality research.
- Terence Tao suggests to have one big problem to go back to when inspired but adhere to (or at least do not neglect) the \publish or perish" model most of the time.

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Research Complexity vs Recognition

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Theory vs Simulation

- A good theory should be able to besimulated.
- A good simulation may change/iron out a theory .
- Cedric Villani thinks that the process can go back and forth until the picture becomes clearer.

Memorable Events In uencing Research

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The Rise of Big Data

- What is Big Data : lots of anecdotal claims about how big is Big Data [9, 2, 16] but the term refers more to the concept of \ data cation " (increase in sizex better con dence interval but rather change in perspective).

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- The Flash Crashes (eg: [29]) calls for a modelling revolution [5, 12, 6] (BU vs. TD): the Brownian motion assumption to model markets is increasingly di cult to defend.

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- Performing a computer tournament with a set of simple xed classic strategies as well as proposing some interesting connection to other seemingly unconnected concepts (Morality and Complexity/Diversity debate),
- Formalizing the methodology for a Particle Filter which aim is to track ecosystems of strategies through time by looking at price dynamics only as well as performing few simulations.

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- Its Portfolio Optimization using several the Bandwise-Gaussian mixture model as well as Deep Learning,
- Reconciling discordant Risk: Anticipative Responsible VaR.

Two simple questions for two simple de nitions

De nition (Top-Down Vs Bottom-Up)

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Can we use this new angle to \solve" the market?

The Scienti c Method for \solving" the market

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Caveat: Is the idea mad, ambitious or both?

The Scienti c Method for \solving" the market

Scienti c Method : A good theory can be simulated but simulations can also help bring intuition on what the theory might be [34].

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Predator/Prey models

biological ecosystemspredator/prey (PP) [35, 8] models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between xˆt•, yˆt• and zˆt• is deterministic :

$$\begin{cases} \frac{dx\hat{}t\bullet}{dt} & ax\hat{}t\bullet \quad bx\hat{}t\bullet y\hat{}t\bullet \\ \frac{dy\hat{}t\bullet}{dt} & cy\hat{}t\bullet \quad dx\hat{}t\bullet y\hat{}t\bullet \quad ey\hat{}t\bullet z\hat{}t\bullet \\ \frac{dz\hat{}t\bullet}{dt} & fz\hat{}t\bullet \quad gy\hat{}t\bullet z\hat{}t\bullet \end{cases}$$

Predator/Prey models

We can make the hypotheses that the economical cycles or oscillation in prices are due to the same type of disruptions that can occur in biological ecosystemspredator/prey (PP) [35, 8] models (a, b, c, d, e, f and g are rate of growth or predation). The relationship between xˆt•, yˆt• and zˆt• is deterministic :

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$$\begin{cases} \frac{dx(t)}{dt} & ax(t) \quad bx(t)y(t) \\ \frac{dy(t)}{dt} & cy(t) \quad dx(t)y(t) \quad ey(t)z(t) \\ \frac{dz(t)}{dt} & fz(t) \quad gy(t)z(t) \end{cases}$$

Answering if an ecosystem (or by extension financial market) composed of 3 strategies is stable would come to studying the Jacobian matrix J [8]. If if all eigenvalues of $J(x; y; z)$ have negative real parts then our system is asymptotically stable. Though simplistic, the model can easily be expanded to more complex ecological niches.

$$J(x; y; z) \quad \begin{pmatrix} a \quad by & xb & 0 \\ yd & c \quad dx \quad ez & ye \\ 0 & zg & f \quad gy \end{pmatrix}$$

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The methodology in ED is interesting because the strategies are both systematic & interacting with each other (like it is the case in algo trading).

A rst application in Economics: Minority Game

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Physical Laws can emerge from these simple rules. We can observe that σ^2/N is only a function of $\alpha = 2^m/N$ which considering the complexity of the interactions between the set of agents can be quite remarkable.

Criticism : Is this realistic for Economics? Maybe, but not for algorithmic trading (eg: TF strategy in a TF concentrated ecosystem)?

Bringing these Ideas into Electronic Trading

As ML \translation" of ED and PP models, we have Generative Adversarial Networks (GANs) [14], introduced in 2014, usually involve a system oftwo neural networks competing in a zero-sum game settings. This process continues as long as needed since the lack ofdata is no longer a problem.

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The communication tool is the order book and the game is not zero-sum game in our research.The strategies are in High Frequency Financial Funnel (HFFF) format [23].

High Frequency Financial Funnel & Classic Strategies

HFFF can model nancial strategies :

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Architecture Complexity and strategy sophistication explains the incentive for Deep Learning (DL).
Paradoxically we witness potential for regularization as the network becomes more complex.

Genetic Algorithm & NN Complexity

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NN depth & breadth should contribute in increasing the learning potential .

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Question: Can we make an analogy to the predator prey ecosystem? Do we get similar behaviour as the Lotka-Volterra equations [35, 19]?

The Path of Interaction

Answer: not quite but there are few interesting links (exponential grow of the smaller prey/self fulfilling strategies such as TF) but they are many issues (classification, timescale etc...).

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The genetic algorithm presented in the previous slide creates complications (classi cation issues). This pushed us to study the bottom-up approach using concepts taken from evolutionary dynamics and created the the concept of Path of Interaction : table of 7 rows documenting the interaction's details (eg: table above).

The Path of Interaction

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Strategy	seed‘	TF1	TF2	TF1	TF2
Iteration	0		1		2
Signal	N/A	+1	+1	+1	+1
OB	${}_{0}P^{1;1;1;1}$	${}_{0;0}P^{1;1;1}$	${}_{0;0;0}P^{1;1}$	${}_{0;0;0;0}P^{1}$	${}_{0;0;0;0;0}P$
Last Price	100	101	102	103	104
OI	1	1	2	3	4
Price	1	1	1	1	1
P&L	0; 0		1; 0		2; 1

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Sequential Monte Carlo

Sequential Monte Carlo (SMC) methods [10, 11, 18], also know as Particle Filter have emerged as a fashionable tool to track scenarios in the last 15 years [31, 32]. They are the sequential analogue of Markov Chain Monte Carlo (MCMC) methods and similar to importance sampling methods.

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The aims of the PF is to estimate the sequence of hidden parameters (eg: the frequencies of certain types of strategies), based on an indirect observations (eg: the uctuations of the market).

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Each deterministic path corresponds to a sequence of interaction of several strategies for which the sequence and the P&Ls can be traced through our SMC methods by looking at the market price only.

PF assigns a probability for each ecosystem scenario

We have recorded 15 di erent scenarios (ecosystem history) for the sake of this presentation, all of which are clearly detected after the 11th iteration [27].

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Model Assuming Data

Definition (Correlation Model)

On a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ the Correlation model is given by the set of two SDE's:

$$
\begin{aligned}
dX(t) &= \mu X(t)\,dt + \sigma X(t)\,dW(t), \\
dY(t) &= \mu Y(t)\,dt + \sigma Y(t)\,d\tilde{W}(t), \qquad (1) \\
d\langle W, \tilde{W} \rangle_t &= \rho\,dt, \qquad (2)
\end{aligned}
$$

where $\mu \in \mathbb{R}$, $\sigma > 0$ are drift and diffusion coefficients of asset price, $\tilde{W}(t)$ and $W(t)$ are two correlated Brownian motions with constant correlation coefficient $\rho \in (-1, 1)$.

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Remark: few assumptions need to be fulfilled. Variance must remain constant (homoscedasticity) and the returns independent. If not instantaneous measured correlation is misleading with respect to long term correlation:

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d`W ; W̃ e_t   dt ;   (2)

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- Bottom Right Figure ^ 1.

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Data Disagrees with Assumptions

Data violates assumptions of the classic model (2):

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- directly (this is not new: the returns are not iid and the variance is not constant),
- or indirectly (Figure on left hand side). Implied correlation is \marked", the same way implied volatility is. The ratio of two closely related underliers exhibits mean reversion in the minds of the traders risk managing these products.

Data Reassuming the Model

Intuitive De nition : Cointelation is a portmanteau neologism in nance, designed to signify a hybrid method between betweencointegration and correlation models (Data-Driven adjustment to classic nancial math models: data \reassuming the model").

Data Reassuming the Model

De nition (Cointelation Model)

On a ltered probability space ˆ ;F ;ˆF •ˆ tC0• ; P• the Cointelation model is given by the SDE's:

dX ˆt• X ˆt•dt X ˆt•dW ˆt•;

dY ˆt• ˆX ˆt• Y ˆt•• dt Y ˆt•dW̃ ˆt•;

d`W ; W̃ e$_t$ dt ;

with >R, A 0, A 0 are drift and di usion coe cients of asset price X , - the rate of mean reversion, W̃ ˆt• and W ˆt• are two correlated Brownian motions with constant correlation coe cient > 1; 1 . The process ˆX ˆt•• $_{tC0}$ is called the leading process, ˆY ˆt•• $_{tC0}$ the lagging process.

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Remark: We note that setting 0 yields the classic correlation model. Conversely, setting 0 yields the cointegration model.

Interesting Property: the Inferred Correlation

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Remark: Cointelation model can hit the whole correlation spectrum depending on timescale.

Interesting Property: the Inferred Correlation

De nition (Inferred Correlation)

Considering, the dynamics of equation (3), the Inferred Correlation formula is given by (3).

$$‡ \quad ˆ1 \quad • \; 1 \quad \exp ˆ \quad ˆ \quad 1•• \tag{3}$$

where $‡ \quad E \sup_{0@B} \; t \; , \quad > Z^{‡}, \quad > 0; 1$ and constant.

Remark: Cointelation model can hit the whole correlation spectrum depending on timescale.

Application: Socially Responsible Finance

Socially Responsible & Consumer Finance is a wave of quant nance that gains momentum after each crisis but quickly runs unfortunately out of fuel:

- Situation : Equity/Commodities salesman trying to convince clients who have a portfolio in the Commodities/Equities asset class to diversify by buying salesman's products.
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We consider the portfolio of:

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$w_t^{‡}$ max 2 E ˆ r_p • 2ˆ r_p • 1 šU Š (X_t Y_t)/Y_t • UB Ÿ
(under max: hˆt•)

sup E 1/ ˆ V ˆ T •• 1 šU Š (X_t Y_t)/Y_t • UA Ÿ
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$$w_t^{\ddagger} \quad \max_{h\hat{}t\bullet} 2 \; E\hat{}r_p\bullet \quad {}^{2}\hat{}r_p\bullet \; 1_{šU\,Š\frac{X_t \quad Y_t}{Y_t}\bullet UB \quad Ÿ}$$

$$\sup_{\hat{}t\bullet >A\hat{}0;v_0\bullet} \quad E \; \frac{1}{}\hat{}V \;\; \hat{}T\bullet\bullet \quad 1_{šU\,Š\frac{X_t \quad Y_t}{Y_t}\bullet UA \quad Ÿ}$$

The red part does not have a closed for solution: the pure Classic Financial Mathematics does not have a solution.

Portfolio Optimization: ML/FM Hybrid Method

The way to handle the red part part is to consider the function $G(t;v;s)$ such that $G \in C^{1;2}(Q)$, the Hamilton-Jacobi-Bellman (HJB) equation corresponding to stochastic control problem is

$$\frac{\partial G}{\partial t}(t;v;s) \quad \sup L \quad G(t;v;s) \quad 0; \tag{4}$$

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- Strategy S in which we are short both X and Y at time t within bands $a_i; b_i$, $i > N$, and with P&L $V_{a_i; b_i; t}$.

Remark: We can have as many bands (strategies) as we have weight proportions.

Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left gure)

Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left gure) in rolling contract form

Anomaly Detection & Volatility Surface de-Arbitraging

Normalizing the data coming from the markets (left gure) in rolling contract form into a coherent xed pillars implied volatility surface (bottom right gure) presents challenges.

Anomaly Detection & Volatility Surface de-Arbitraging

The classic arbitrage conditions (butter y & calendar spread) have been replaced with more elegant models (e.g. K ; ¦ T ; S T @$_K$ ^{2}ˆ K ; T •S B 4) that were nevertheless not su cient and the arrival of Big Data in the wings exposed these limitations in the Financial Mathematics models.

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Figure: Vogt's total variance example verifying bˆ1 S S•B $\frac{4}{T}$ (left gure: with the x axis being the log-moneyness and the y axis being the implied variance) and the corresponding @²K ; K BS ‰ ²ˆK ; T •Ž approximating the (supposed) always positive pdf (right gure: with the x axis being the log-moneyness and the y axis being the non normalized pdf).

Anomaly Detection & Volatility Surface de-Arbitraging

The work of this chapter revolves around anomaly detection in the context of the implied volatility surface trying to use the classic methods: butter y arbitrage (or call spread) in equation (5b) and calendar spread arbitrage in equation (5c) and the more advanced methods such as in equation (5e) (a necessary but not su cient condition).

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solve: ^t ˆ ; d• arg min Q Q CŠ i;t ˆ ; d•• CŠ~t ˆ ; d•• 2 (5a)
´111111111 11111¹11¶
~t ˆ ; d• ¸

subject to: ¦ and ¦ K

CŠK ; 0ˆK ; •• CŠK ; 0ˆK ; •• C 0 (5b)

CŠK ; ; 0ˆK ; •• CŠKe r ; 0ˆKe r ; •• C 0 (5c)

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Big Data Changing the Vanilla Options Landscape

exposed the limitations of both the wings (e.g. SVI) and of the liquidity in the options market,

We saw in the Options market :

- a step back in complexity (from exotics to vanilla) but with more coherent pricing
- a step forward in liquidity modelling

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$$dS_t \quad {}^{o} \quad \overline{v}_t S_t\, dW_t^1; \;\; S_0 > R^{\ddagger} \qquad (6a)$$

$$dv_t \quad \hat{} \quad v_t \bullet dt \quad {}^{o} \quad \overline{v}_t\, dW_t^2; \;\; v_0 > R^{\ddagger} \qquad (6b)$$

The rise of big data:

- exposed the limitations of both the wings (e.g. SVI) and of the liquidity in the options market,
- the need for proxying when the data is scarce.
- the Heston (Stochastic Volatility) model and the local volatility model and the need for harmonizing these two concepts.

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Big Data Changing the Vanilla Options Landscape

Big Data exposed the limitations of the SVI in the wings and the subprime crisis of 2007 exposed the need to incorporate liquidity directly in the options model.

We define the Implied Volatility surface Parametrization (IVP) split with its mid in equation (7) with the downside transform in equation (7b) enhancing the SVI,

$$\sigma^2_{IVP;o;}(k) = a + b\left(\rho(z_{o;} - m) + \sqrt{(z_{o;} - m)^2 + \sigma^2}\right) \qquad (7a)$$

$$z_{o;} = \frac{k}{1 + \psi_{o;} S(k - m_S)} \qquad (7b)$$

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$$ z_{o;} \quad \frac{k}{1 \;\; 4\$k \;\; mS \; o;} \tag{7b} $$

and its liquidity parameters in equation (8). Parameters represents the Wings Curvature , represents the ATM Spread . The latter two parameters can be de ned in terms of functions to accommodate the position size in which case the market dept is controlled with the parameters.

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$$ z\;; \quad z_{o;} \quad 1 \quad \hat{}p\bullet \tag{8d} $$

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$$ \hat{}p\bullet \quad 0; \quad \hat{}1 \quad 0; \quad \bullet\hat{}1 \quad e \quad p\bullet \tag{8f} $$

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Big Data exposed the limitations of the SVI in the wings and the subprime crisis of 2007 exposed the need to incorporate liquidity directly in the options model. The IVP addresses these two points as well as allow for additional bene ts:

- Proxying
- Backtesting weaponry on complex strategies.

Big Data, Proxying & Handling Dimensionality for Optio

Clustering for Distribution & Regime Change Forecastin

We show how clustering can help enhance MF and therefore the two fields can be apposed instead of opposed in the context of modelling risk factors (RF) which behave elements of mean reversion (Spread, Options RF). More specifically we look at how we can free oneself with the assumptions of SDEs to construct a general clustering methodology. This can allow us to construct concepts like the Anticipative VaR (a leading regime change) as opposed to Responsive VaR (a lagging regime change).

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In the left gure, we apply clustering in order to classify dynamic zones in which the returns act di erently . For example when the underlier is signi cantly above its mean, the forecasted distribution is normally distributed with however a negative mean (vice versa when the underlier is below its historical mean). The distribution is symmetric when at the long term mean.

Clustering for Distribution & Regime Change Forecastin

In order to Reconcile discordant instructions of our regulators to create a risk measure which isresponsive but stable at the same time we propose the Responsible VaR, a risk measure responsive on the upside but stable on the downside. We give a couple of examples (gures on the left) of complex portfolio (straddle) backtests in which we modify the   to control the stability on the downside.

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$$S_{a}^{t} \; p_t \hat{} x \bullet dx \tag{9a}$$

$$\tilde{}_0 \quad 0 \tag{9b}$$

$$\tilde{}_t \quad \max Š_t ; \; \tilde{}_{t} \; 1 \quad \hat{} 1 \quad \bullet \; _t \bullet \tag{9c}$$

$$1 \quad S_{t}^{a} \; p_t \hat{} x \bullet dx \tag{9d}$$

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S a t pt ˆ x • dx (9a)

˜0 0 (9b)

˜t max Š t ; ˜t 1 ˆ 1 • t • (9c)

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˜0 0 (9e)

˜t min Š t ; ˜t 1 ˆ 1 • t • (9f)

In order to Reconcile discordant instructions of our regulators to create a risk measure which isresponsive but stable at the same time we propose the Responsible VaR, a risk measure responsive on the upside but stable on the downside. We give a couple of examples (gures on the left) of complex portfolio (straddle) backtests in which we modify the to control the stability on the downside.

$$S_{a}^{t}\ p_t\,\hat{}\,x\bullet dx \tag{9a}$$

$$\tilde{}_0 \quad 0 \tag{9b}$$

$$\tilde{}_t \quad \max Š_{t}\ ;\ \tilde{}_{t}\ _1 \quad \hat{}1 \quad \bullet\ _t \bullet \tag{9c}$$

$$1 \quad S_{t}^{a}\ p_t\,\hat{}\,x\bullet dx \tag{9d}$$

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- Finally we looked at tracking methods using MTT .

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- Is TF the most \moral" strategy ? TF is similar to the TFT (it replicates but adapts).

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- Additional Liquidity issues for Implied Volatility (the ATM Bid Ask is asset class sensitive) which makes the model less universal.

Future Research (Part II: Apposition)

Harmonizing Stochastic & Local Volatility : We have seen that both the Heston and SVI models are popular in the industry and converge asymptotically to each other [13]: see Equation (10). Are their limitations linked ?

$$dS_t \quad {}^{o}\sqrt{v_t}\, S_t\, dW_t^1; \qquad S_0 > R^{\ddagger} \tag{10a}$$

$$dv_t \quad \hat{} \quad v_t \bullet dt \quad v_t^{\frac{1}{2}}\, dW_t^2; \qquad v_0 > R^{\ddagger} \tag{10b}$$

$$d\, aW^1; W^2 f_t \quad dt; \tag{10c}$$

$$v\hat{}k;t\bullet \quad a \quad b \quad \hat{}k \quad m\bullet \quad \gg \overline{\hat{}k \quad m\bullet^2 \quad {}^2} \tag{10d}$$

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IVP and Assumed Correlation of Equation (11) the answer?

$$dS_t \quad {}^{o}\overline{v_t} S_t \, dW_t^1; \qquad S_0 > R^{\ddagger} \tag{11a}$$

$$dv_t \quad \hat{} \quad v_t \bullet dt \quad {}^{o}\overline{v_t} \, dW_t^2; \qquad v_0 > R^{\ddagger} \tag{11b}$$

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$$\hat{} t; S_t \bullet \quad \hat{} t \bullet \quad \hat{} t \bullet \quad \hat{} t \bullet \quad 1 \quad \exp \hat{} \quad \hat{} t \bullet S S_t \quad K \, S \bullet \tag{11d}$$

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Closing Statement with Q&A

{ Russell L. Acko

[1] a quote that is generally attributed to George Box
[2] to mean, less useful for practitioners.

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To that extend we know that all models are wrong, but some are useful[1] and in that spirit we have arguably done the right thing wronger [2] in the 1st part of the thesis but the wrong thing righter in the 2nd part.

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